X-711-70-174
PREPRINT

NASA TMX-70572

OPTICAL ASPECT SYSTEM FOR THE SMALL SCIENTIFIC SATELLITE (S^3-A)

E. JOHN PYLE

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GSFC — GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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SECTION I

INTRODUCTION

The S^3-A is a spin stabilized satellite which will study particles and fields within the inner magnetosphere. It will be placed into an equatorial orbit with a near-earth perigee and an apogee altitude of approximately five earth radii. The scientific objectives of the mission can best be accomplished if the spacecraft spin axis remains in the equatorial plane. This paper describes the Optical Aspect System designed to measure the attitude parameters necessary for the determination of spin axis orientation of S^3-A.

The theoretical considerations of the rotational motion of a rigid body are presented along with the equations necessary for the solution of the aspect problem. Two detectors are used to obtain angular information from the sun and the sunlit earth. A digital solar sensor which performs an optical analog-to-digital conversion is used to obtain information on the elevation and azimuth angles of the solar disk. Information is obtained from the sunlit earth by the use of a visible horizon detector which responds to the visual discontinuity caused by the sunlit earth against the darkness of interplanetary space. The logic system necessary to collect, process and store information from these two detectors is presented and discussed.

SECTION II

THEORETICAL CONSIDERATIONS OF ASPECT DETERMINATION

Energy and Momentum Considerations

In the solution to the problem of force free motion of a rigid body, a point on the body can be defined in terms of the body fixed axes (x, y, z). For the purpose of defining the orientation of the body fixed axes relative to a set of nonrotating reference axes (X, Y, Z), Euler introduced three independent angles (θ, ψ, θ). These Eulerian angles are shown in Figure 1 and are defined by three successive rotations performed in a specific sequence. The sequence is started by: (1) rotation of the initial system of axes (X, Y, Z) counterclockwise about the Z-axes through an angle ϕ, producing the intermediate set of axes (ξ', η', ζ'). (2) Next, this intermediate set of axes is rotated counterclockwise through an angle θ about the ξ'-axis producing a second intermediate set of axes (ξ, η, ζ). The ξ'-axis is also called the line of nodes. (3) Finally, the (ξ, η, ζ) axes are rotated counterclockwise about the ζ-axis through an angle ψ, forming the (x, y, z) set of axes. Hence, the Eulerian angles (ϕ, ψ, θ) completely describe the orientation of the (x, y, z) system with respect to the (X, Y, Z) coordinate system. Let the (x, y, z) coordinate system be aligned with the three principle moments of inertia of the satellite I_1, I_2, and I_3. Hence, the (x, y, z) system is fixed in the rotating satellite. The instantaneous values of momentum about the x, y, and z axes are, respectively:

$$p_x = I_1 \omega_x \tag{1}$$

$$p_y = I_2 \omega_y \tag{2}$$

$$p_z = I_3 \omega_z \tag{3}$$

where ω_x, ω_y, and ω_z are the instantaneous values of angular velocity about the x, y, and z axes. The instantaneous values of both the momentum (p_x, p_y, p_z) and the angular velocity ($\omega_x, \omega_y, \omega_z$) about the x, y, and z axes can be expressed in terms of the momentum vector (see Appendix B).

$$p_x = L \sin\theta \sin\psi \tag{4}$$

$$p_y = L \sin\theta \cos\psi \tag{5}$$

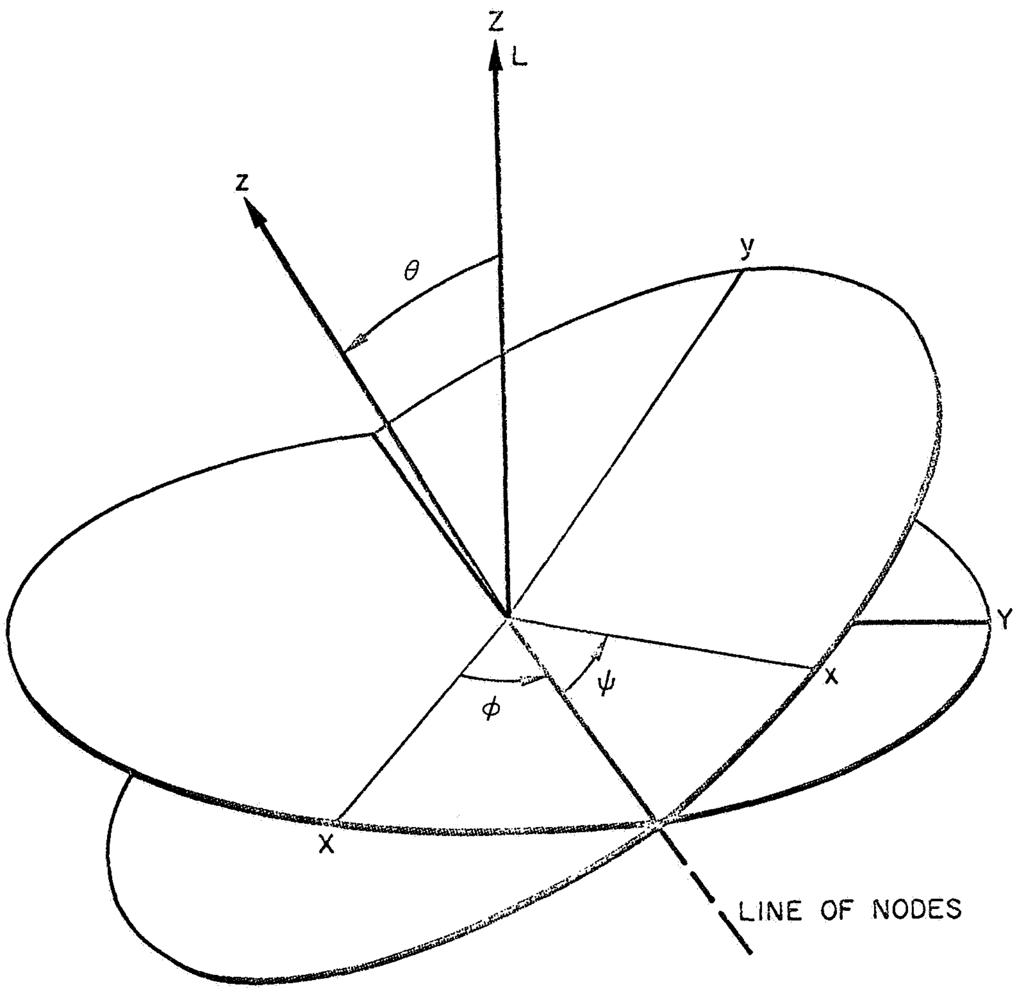

Figure 1. Euler's Rotation Angles

$$p_z = L \cos\theta \tag{6}$$

$$\omega_x = \dot{\phi} \sin\psi \sin\theta + \dot{\theta} \cos\psi \tag{7}$$

$$\omega_y = \dot{\phi} \cos\psi \sin\theta - \dot{\theta} \sin\psi \tag{8}$$

$$\omega_z = \dot{\phi} \cos\theta + \dot{\psi} \tag{9}$$

Substitution of Equations (4), (5), (6), (7), (8), and (9) into Equations (1), (2), and (3) yields:

$$p_x = L \sin\psi \sin\theta = I_1 (\dot{\phi} \sin\psi \sin\theta + \dot{\theta} \cos\psi) \tag{10}$$

$$p_y = L \cos\psi \sin\theta = I_2 (\dot{\phi} \cos\psi \sin\theta - \dot{\theta} \sin\psi) \tag{11}$$

$$p_z = L \cos\theta = I_3 (\dot{\phi} \cos\theta + \dot{\psi}) \tag{12}$$

Multiplying Equation (10) by $\sin\psi$ and Equation (11) by $\cos\psi$ and adding the results gives:

$$L \sin\theta = \left(I_1 - I_2\right) \dot{\theta} \cos\psi \sin\psi + \dot{\phi} \sin\theta \left(I_1 \sin^2\psi + I_2 \cos^2\psi\right) \tag{13}$$

Multiplying Equation (10) by $\cos\psi$ and Equation (11) by $\sin\psi$ and subtracting gives:

$$0 = \dot{\phi} \cos\psi \sin\psi \sin\theta \left(I_1 - I_2\right) + \dot{\theta} \left(I_1 \cos^2\psi + I_2 \sin^2\psi\right) \tag{14}$$

If the satellite is assumed to be dynamically balanced about the z-axis, then $I_1 = I_2$. Making this assumption and substituting into Equations (13) and (14):

$$L = I_1 \dot{\phi} \qquad \text{(from equ. 13)} \tag{15}$$

$$I_1 \dot{\theta} = 0 \qquad \text{(from equ. 14)} \tag{16}$$

Since $I_1 \neq 0$, $\dot{\theta}$ must be zero and it follows that θ is independent of time.

Substitution of Equations (15) and (16) into (12) yields:

$$p_z = I_1 \dot{\phi} \cos\theta = I_3 (\dot{\phi} \cos\theta + \dot{\psi}) \tag{17}$$

Solving for $\dot{\psi}$, the angular velocity of the satellite about the satellite z-axis:

$$\dot{\psi} = \frac{(I_1 - I_3)\dot{\phi}\cos\theta}{I_3} \tag{18}$$

Solving for $\dot{\phi}$, the angular velocity of the satellite z-axis about the momentum vector or more precisely, the angular velocity of the line of nodes.

$$\dot{\phi} = \frac{I_3 \dot{\psi}}{(I_1 - I_3)\cos\theta} \tag{19}$$

$\dot{\phi}$ will be referred to as the precession rate and $\dot{\psi}$ will be termed the spin rate. This is to be distinguished from the apparent rotation rate of the satellite with respect to a fixed external point. This apparent rotation rate has an average value of $(\dot{\phi} + \dot{\psi})$. θ is the precession cone half angle and $\dot{\theta}$ is the rate of change of the precession cone half angle. If $I_1/I_3 < 1$, θ tends towards zero and the precession coning will damp out in time. Since it is usually desirable for the satellite to rotate about the z-axis, most spin stabilized satellites are balanced so that the z-axis coincides with the largest moment of inertia. The rest of this text will deal only with this case, resulting in the fact that $\theta = 0$. It should not be assumed that zero precession implies that $\dot{\phi}$ goes to zero. Equation (19) states that when $\theta = 0$, $\dot{\phi} = I_3 \dot{\psi}/(I_1 - I_3)$.

Equations Determining Satellite Aspect

Figure 2 shows the spacecraft momentum vector relative to the sun and zenith vector on the celestial sphere. The zenith vector is defined as a vector

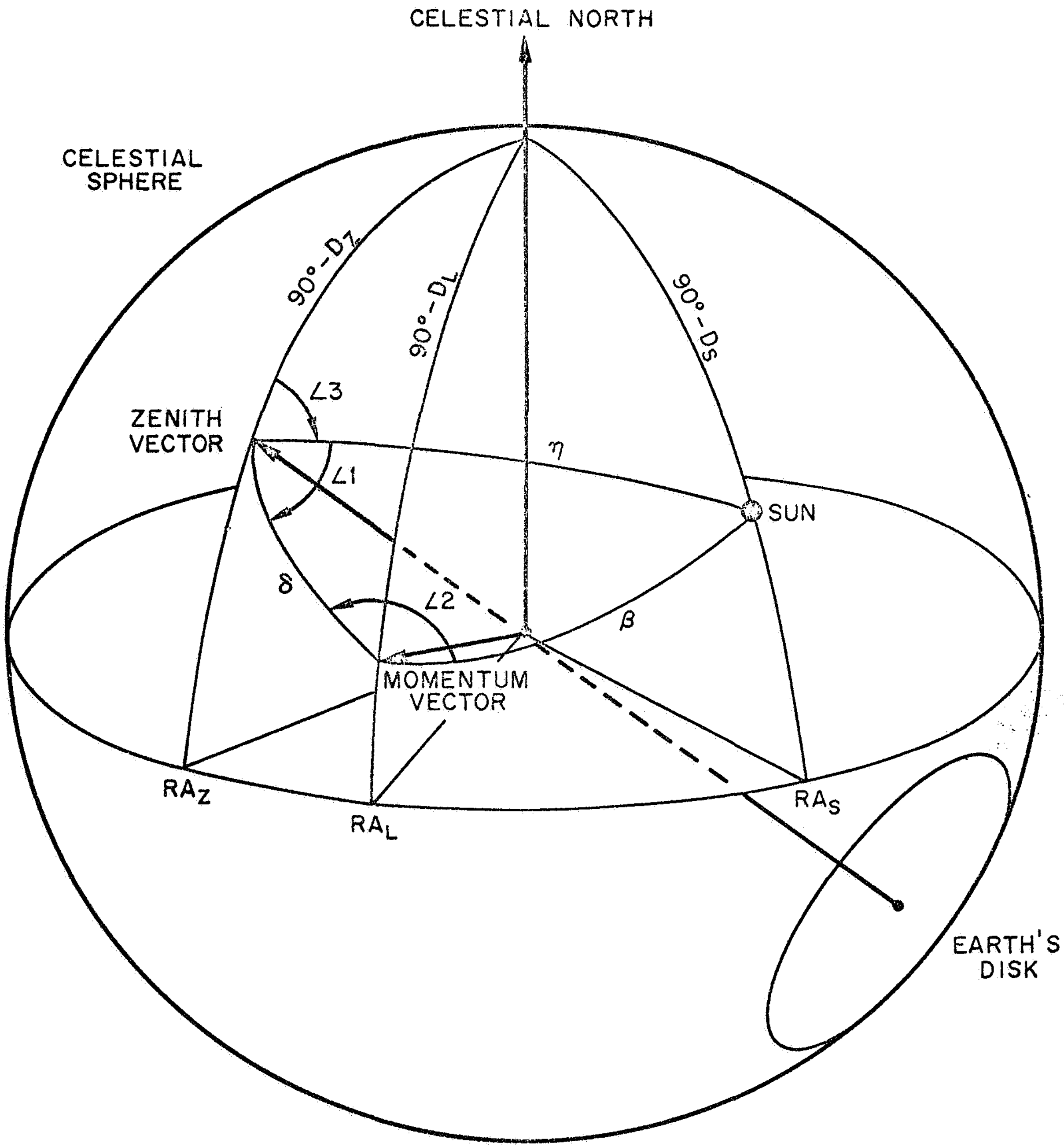

Figure 2. Spacecraft Momentum Vector Relative to the Sun and the Zenith Vector

from the center of the earth through the center of mass of the spacecraft. The center of the earth is located on the celestial sphere at (RA_z + 180°) and ($-D_z$), where RA_z is the right ascension and D_z the declination of the zenith vector. The great circle arc from the sun to the zenith vector is eta (η). By applying the laws of sines and cosines to the spherical triangles in Figure 3, the equations for the values of eta (η) and the angles $\angle 1$ and $\angle 3$ may be written.

$$\cos\eta = \sin D_z \sin D_s + \cos D_z \cos D_s \cos\left(RA_s - RA_z\right) \qquad 0 \le \eta < 180^\circ$$

$$\cos\angle 1 = \frac{\cos\beta - \cos\eta\cos\delta}{\sin\eta\sin\delta}$$

$$\sin\angle 1 = \frac{\sin\beta\sin\angle 2}{\sin\eta}$$

$$\cos\angle 3 = \frac{\sin D_s - \cos\eta\sin D_z}{\sin\eta\cos D_z}$$

$$\sin\angle 3 = \frac{\sin\left(RA_s - RA_z\right)\cos D_s}{\sin\eta}$$

Beta (β) is the angle between the sun and the momentum vector (spin axis) and delta (δ) is the angle between the zenith and momentum vectors. Now using the values of $\angle 1$ and $\angle 3$ the right ascension (RA_L) and declination (D_L) of the momentum vector may be determined.

$$\sin D_L = \sin D_z \cos\delta + \cos D_z \sin\delta\cos\left(\angle 3 + \angle 1\right) \qquad -90^\circ < D_L \le 90^\circ$$

$$\cos\left(RA_L - RA_z\right) = \frac{\cos\delta - \sin D_z \sin D_L}{\cos D_z \cos D_L}$$

$$\sin\left(RA_L - RA_z\right) = \frac{\sin\left(\angle 1 + \angle 3\right)\sin\delta}{\cos D_L}$$

$$RA_L = RA_z + \left(RA_L - RA_z\right)$$

Therefore, to determine the right ascension and declination of the spacecraft's momentum vector, several quantities must be known. The right ascensions and declinations of the zenith vector (RA_z, D_z) and the sun (RA_s, D_s) are accurately known from earth based information. Beta (β), delta (δ) and $\angle 2$ must be determined on board the spacecraft. Considering Figure 2, it is clear that

$$\angle 2 = (\dot{\phi} + \dot{\psi}) \Delta t_c \pm 180°$$

where $(\dot{\phi} + \dot{\psi})$ is the apparent rotation rate of the spacecraft and Δt_c is the time required for the reference plane to move from the sun to the earth's center. Therefore, it is the measurement of beta (β), delta (δ), and $(\dot{\phi} + \dot{\psi}) \Delta t_c$ that the aspect system must perform.

SECTION III

DIGITAL SOLAR SENSOR

Introduction

The digital solar sensor measures the angle of incident sunlight with respect to the sensor z-axis and expresses this angle as a digital number. The incident, sunlight, passing through a slit on the top of a quartz block, is screened by a Gray-coded pattern on the bottom of the block to either illuminate or not illuminate each of the photocell detectors. The angle of incidence determines which combination of photocells is illuminated. The solar sensor also includes a command slit which is mounted perpendicular to the Gray-coded reticle. If the sensor is rotated about a vertical axis along the command slit, the field of view of the two slits will sweep out a solid angle. When the plane containing the command slit passes across the solar disk, one or more of the photocells will be illuminated. The time that this illumination occurs provides a measurement of the azimuth angle of the sun. The particular combination of photocells which is illuminated provides a digital measurement of the elevation angle of the sun in sensor coordinates.

The digital solar sensor has features which make it more desirable than the analog types of solar sensors. It is not subject to errors introduced by earth shine and there are no components which can drift. The sensor requires no in-flight calibration. The digital sensor is light in weight and requires only the amount of power needed to drive the output load.

Measurements Using a Digital Solar Sensor

Calculation of β—To make the measurements necessary to determine the motion of a spinning satellite, assume that a digital solar sensor is mounted on the satellite with the Z-axis of the sensor parallel to the Z-axis of the rotating satellite. Then, as the satellite rotates, the field of view of the sensor will be a fan which sweeps across the sky, which is the celestial sphere. If a bright object such as the solar disk should be in a portion of the celestial sphere swept out by the sensor's field of view, a digital number, defining the angle between the sensor's Z-axis and the sun vector would be placed in storage. This angle between the sensor's Z-axis and the sun vector, measured by the digital sun sensor, will be referred to as beta (β).

Determination of $\dot{\phi}$ and $\dot{\psi}$ —Having determined β we proceed to determine the integrals of motion, $\dot{\phi}$ and $\dot{\psi}$. When $\theta = 0$ (no precession) the apparent rotation velocity of the satellite is $\dot{\phi} + \dot{\psi}$. If T_0 and T_1 represent two different times at which the solar disk appeared in the command slit of the digital solar sensor and N_r is the number of times the solar disk appeared in the command slit between T_0 and T_1, then the average period (T_{ave}) is given by:

$$T_{ave} = \frac{T_1 - T_0}{N_r}$$

and

$$2\pi N_r = (\dot{\phi} + \dot{\psi}) (T_1 - T_0)$$

Combining Equations (18) and (19) with the above expressions we have:

$$\dot{\phi} = \frac{2\pi N_r}{(T_1 - T_0) \left[1 + \frac{(I_1 - I_3) \cos\theta}{I_3}\right]}$$

$$\dot{\psi} = \frac{2\pi N_r}{(T_1 - T_0) \left[1 + \frac{I_3}{(I_1 - I_3) \cos\theta}\right]}$$

and since $\theta = 0$

$$\dot{\phi} = \frac{2\pi N_r}{(T_1 - T_0) \left(1 + \frac{I_1 - I_3}{I_3}\right)}$$

$$\dot{\psi} = \frac{2\pi N_r}{(T_1 - T_0) \left(1 + \frac{I_3}{(I_1 - I_3)}\right)}$$

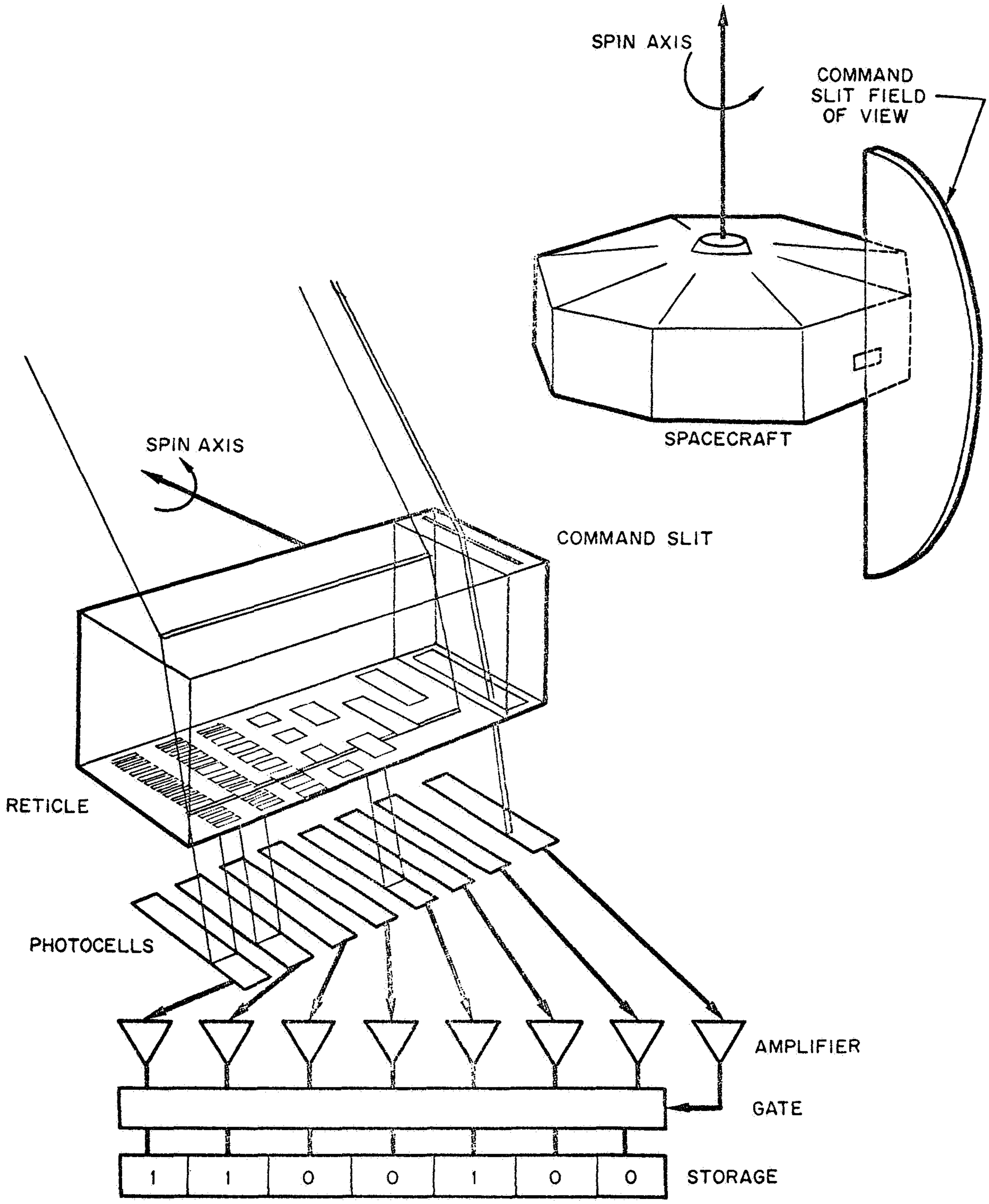

Figure 3. Schematic Representation of a Digital Solar Sensor

SECTION IV

EARTH HORIZON DETECTOR

Introduction

Thus far all the measurements have been made on the sun. Since the sun is considered a point source at infinity, there is an axis of symmetry from the center of gravity of the satellite to the sun. Another inertial reference point is necessary in order to eliminate the ambiguity caused by this symmetry. For this purpose, the satellite is also equipped with an earth horizon detector sensitive to visible light.

The horizon detector consists of a simple telescope and lens. The detector element is a photodiode which is placed at the focal point of the lens. The field of view of the horizon detector is a pencil beam approximately three degrees in diameter. If the detector is mounted at an angle gamma (γ) from the spin axis of a rotating satellite, the field of view of the detector will transverse the surface of a cone whose half angle is equal to gamma (γ). When the field of view scans the discontinuity caused by the sunlit earth against the dark background of interplanetary space, this detector produces a positive output pulse. The time at which these pulses occur provides azimuth information on the earth's angular distance from the sun.

Measurements Using a Horizon Detector

The information contained in the relative position of the detector output pulses to the command slit crossing of the sun provides a measure of the inclination of the satellite zenith vector to the spin axis. Consider the celestial sphere shown in Figure 4 with the spacecraft located at the center of the sphere. The arc length χ_1 is the smaller of the two possible great circle arc lengths from the sun to the horizon. From spherical trigonometry it follows that:

$$\cos \chi_1 = \cos\beta \cos\gamma + \sin\beta \sin\gamma \cos A$$

There are two values of χ_1 which satisfy this equation. This ambiguity can be removed by considering the magnitude of the angle A.

$$\text{if} \quad \pi - A > 0 \ , \quad \text{then} \quad 0 \leq \chi_1 < \pi$$

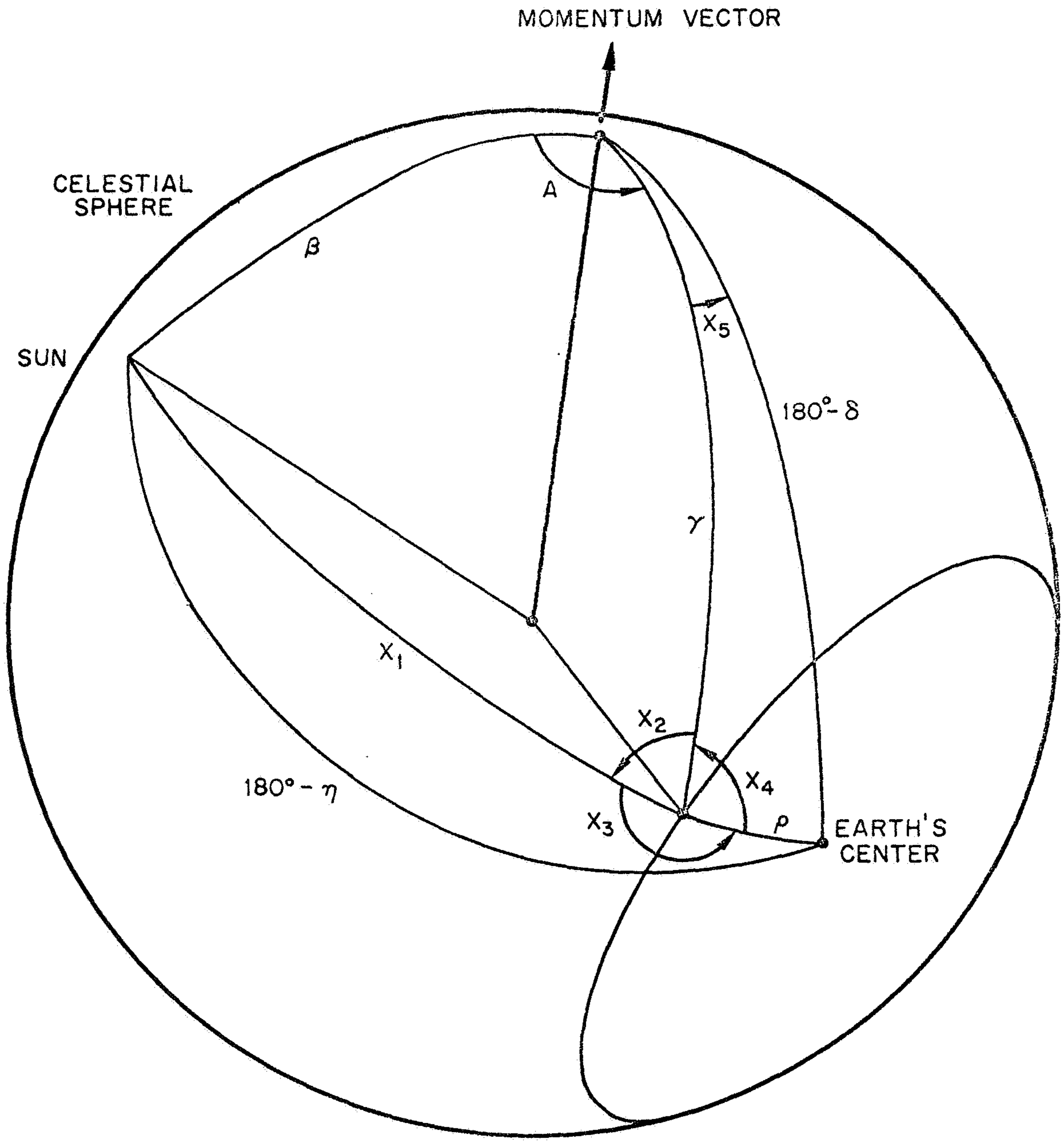

Figure 4. Celestial Sphere Representation of the Single Horizon Solution for Delta

$$\text{if} \quad \pi - A < 0 , \quad \text{then} \quad \pi < \chi_1 \leq 2\pi$$

$$\text{if} \quad \pi - A = 0 , \quad \text{then} \quad \chi_1 = \beta + \gamma$$

The angle A corresponds to the relative position of the earth's horizon with respect to the command slit crossing of the solar disk. Hence χ_1 is uniquely determined and:

$$\sin \chi_1 = \sqrt{1 - \cos^2 \chi_1}$$

Now applying the cosine and sine laws of spherical trigonometry to the triangle containing the angle χ_2:

$$\cos \chi_2 = \frac{\cos \beta - \cos \chi_1 \cos \gamma}{\sin \chi_1 \sin \gamma}$$

$$\sin \chi_2 = \sin \beta \frac{\sin A}{\sin \chi_1}$$

Therefore χ_2 is also uniquely determined.

Let R equal the mean radius of the earth and h be the height of the satellite above the earth then:

$$\sin \rho = \frac{R}{R + h} \qquad 0 \leq \rho < 90^\circ$$

$$\cos \chi_3 = \frac{- \cos \eta - \cos \chi_1 \cos \rho}{\sin \chi_1 \sin \rho}$$

Now again there are two values of χ_3 that satisfy this equation. Also from Figure 5

$$\chi_4 = 2\pi - (\chi_2 \pm \chi_3)$$

and χ_4 will have two possible values corresponding to the two values of χ_3. Also from Figure 5

$$\cos\delta = -\cos\rho\cos\gamma - \sin\rho\sin\gamma\cos\chi_4 \qquad 0 \leq \delta < 180° \qquad (20)$$

and delta (δ) will be double valued.

The information contained in the spacing of the horizon detector output pulses and the relative position of the earth's center from the sun, provide alternate methods of calculating delta (δ), (see Appendix C). The equations expressing these results are presented here for convenience.

$$\cos\delta = \frac{-de \pm f\sqrt{e^2 + f^2 - d^2}}{e^2 + f^2} \qquad 0 \leq \delta < 180° \qquad (21)$$

where

$$d = \cos\rho$$

$$e = \cos\gamma$$

$$f = \sin\gamma\cos\left(\frac{\pi\Delta t_a}{T}\right)$$

$$\cos\delta = \frac{rs \pm \mu\sqrt{s^2 + \mu^2 - r^2}}{s^2 + \mu^2} \qquad 0 \leq \delta < 180° \qquad (22)$$

where

$$r = \cos\eta$$

$$s = \cos\beta$$

$$\mu = -\sin\beta\cos\left(\frac{2\pi\Delta t_c}{T}\right)$$

The two expressions above are based on the assumption that an earth terminator is not transversed as the earth sensor sweeps across the terrestrial disk.

The information which the earth detector must provide to solve Equations (21) and (22) are the apparent width of the earth's disk (Δt_a) and the location of the earth's center from the sun (Δt_c). From Figure 4, Δt_c can be related to the measured angle A by solving the spherical triangle for χ_5.

$$\cos \chi_5 = \frac{\cos \rho + \cos \gamma \cos \delta}{\sin \gamma \sin \delta}$$

$$\sin \chi_5 = \sin \rho \frac{\sin \chi_4}{\sin \delta}$$

$$\chi_5 = \tan^{-1} \left(\frac{\sin \chi_5}{\cos \chi_5} \right)$$

$$\Delta t_c = A + \chi_5$$

The value of Δt_c may also be calculated from:

$$\Delta t_c = A + \frac{\Delta t_a}{2}$$

where Δt_a is the apparent earth width and is measured on board the spacecraft.

Now each of the solutions given in Equations (20), (21) and (22) yield two values of the angle delta. This ambiguity is eliminated by considering the time function of the apparent earth's width with the movement of the spacecraft in its orbit.

SECTION V

ASPECT SYSTEM ELECTRONICS

Data Processing System Interface

The Data Processing System accepts the aspect data through the use of a Interrupt Service Subroutine (I.S.S.). Data processed through an I.S.S. is entered into the data table area of the buffer memory. Associated with each I.S.S. are two data tables which are used alternately for storing and reading out. These data tables, when storing data, are filled backwards, i.e., the first data word is stored in location N, the next in location N-1, etc. When storage location 1 becomes filled, further interrupt requests are ignored until read out from the other data table to telemetry is completed. When this is accomplished, the two data tables are interchanged. Data is read out of the table that was previously used for storage, and interrupt requests are processed and new data is placed in the table that just completed read out.

An I.S.S. request is initiated by the aspect system generating a data ready signal. At the same time, two code bits are also generated by the aspect system and sent to the D.P.S. to identify the entry point into the interrupt service subroutine. When service for the interrupt is completed, the D.P.S. generates a data stored signal which is sent to the aspect system.

If a failure occurs in the buffer memory, the D.P.S. can no longer service the aspect data through the use of the interrupt service subroutine. Therefore, the occurrence of events which generate aspect data, such as the sun and earth sighting, can no longer be recorded by placing the spacecraft clock time in storage in the data tables. This data must now be read out of the aspect system directly by the telemetry format program (P1), which operates in synchronism with the spacecraft clock.

This type of operation places certain restrictions on the aspect system. First, the data is read out of the aspect system in synchronization with the spacecraft clock. This means the aspect system must store the data until the proper read out time. Secondly, the measurement of an aspect event must now be made with respect to some known telemetry time, i.e., measurement of a time interval instead of the instantaneous time the event occurred.

Solar Sensor Electronics

The solar sensor output consists of ten channels of sun information. Nine of these contain angular information, the tenth contains time occurrence or azimuth information.

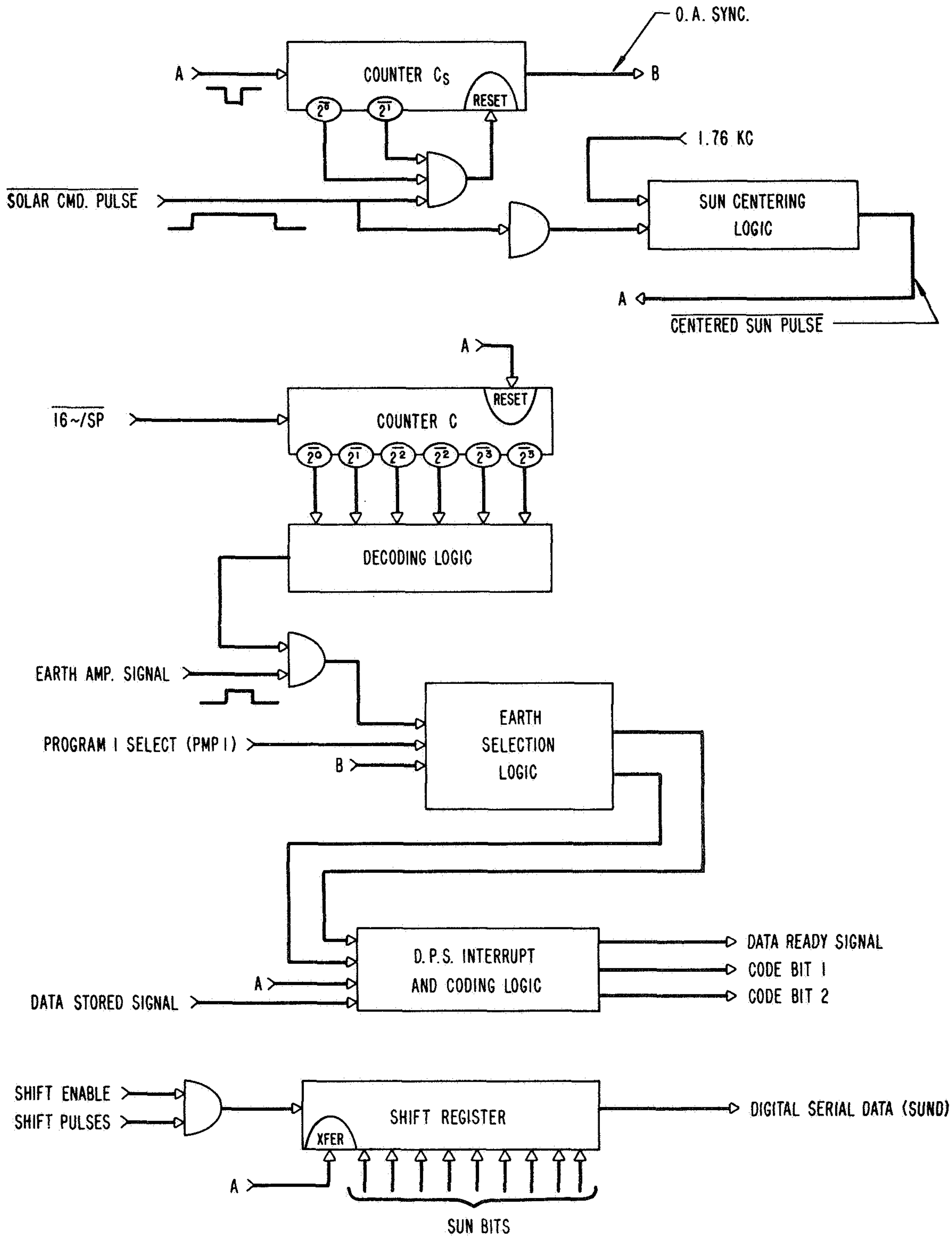

Figure 5. Aspect System Logic: Normal Operation

OPTICAL ASPECT WORD

BYTE N				BYTE N+1				BYTE N+2				BYTE N+3			
IC-11 CODE BIT 1	IC-12 CODE BIT 2	DATA DATA - "1" NO DATA "0"	TABLE TABLE A "0" TABLE B "1"	BIT-1	BIT-2	BIT-3	BIT-4	BIT-1	BIT-2	BIT-3	BIT-4	BIT-1	BIT-2	BIT-3	BIT-4
SUN ANGLE DATA															
1	0	1		SUN BIT 1	SUN BIT 2	SUN BIT 3	SUN BIT 4	SUN BIT 5	SUN BIT 6	SUN BIT 7	SUN BIT 8	SUN BIT 9	0	SEL. SUN I.D. NO EARTH "1" EARTH - "0"	PSEUDO I.D. TRUE SIG. "1" PSEUDO "0"
SUN TIME FINE (SCTF)															
1	0	1		2^9 BIT RATE	2^8 BIT RATE	2^7 BIT RATE	2^6 BIT RATE	2^5 BIT RATE	2^4 BIT RATE	2^3 BIT RATE	2^2 BIT RATE	2^1 BIT RATE	2^0 BIT RATE	2^{-1} BIT RATE	2^{-2} BIT RATE
SUN TIME COARSE (SCTC)															
1	0	1		2^{18} BIT RATE	2^{17} BIT RATE	2^{16} BIT RATE	2^{15} BIT RATE	2^{14} BIT RATE	2^{13} BIT RATE	2^{12} BIT RATE	2^{11} BIT RATE	2^{10} BIT RATE	2^9 BIT RATE	2^8 BIT RATE	2^7 BIT RATE
EARTH "IN" TIME FINE (SCTF)															
0	1	1		2^9 BIT RATE	2^8 BIT RATE	2^7 BIT RATE	2^6 BIT RATE	2^5 BIT RATE	2^4 BIT RATE	2^3 BIT RATE	2^2 BIT RATE	2^1 BIT RATE	2^0 BIT RATE	2^{-1} BIT RATE	2^{-2} BIT RATE
EARTH "IN" TIME COARSE (SCTC)															
0	1	1		2^{18} BIT RATE	2^{17} BIT RATE	2^{16} BIT RATE	2^{15} BIT RATE	2^{14} BIT RATE	2^{13} BIT RATE	2^{12} BIT RATE	2^{11} BIT RATE	2^{10} BIT RATE	2^9 BIT RATE	2^8 BIT RATE	2^7 BIT RATE
EARTH "OUT" TIME FINE (SCTF)															
1	1	1		2^9 BIT RATE	2^8 BIT RATE	2^7 BIT RATE	2^6 BIT RATE	2^5 BIT RATE	2^4 BIT RATE	2^3 BIT RATE	2^2 BIT RATE	2^1 BIT RATE	2^0 BIT RATE	2^{-1} BIT RATE	2^{-2} BIT RATE
EARTH "OUT" TIME COARSE (SCTC)															
1	1	1		2^{18} BIT RATE	2^{17} BIT RATE	2^{16} BIT RATE	2^{15} BIT RATE	2^{14} BIT RATE	2^{13} BIT RATE	2^{12} BIT RATE	2^{11} BIT RATE	2^{10} BIT RATE	2^9 BIT RATE	2^8 BIT RATE	2^7 BIT RATE

Figure 6. Optical Aspect Word Format: Buffer Memory

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Amplification of the output signals of the nine angular information channels produces a plus voltage when the channel is excited, i.e., sunlight present on the detector. These amplified signals are then placed in storage and, at the proper time, shifted into the buffer memory data tables.

The amplified azimuth information, referred to as the solar command pulse, occurs when the sun is in the field of view of the sensor command slit. The width of this pulse is a function of the spin rate and the command slit field of view. To eliminate these undersirable effects, a centered sun pulse is generated at the midpoint of the solar command pulse (sun centering module).

The trailing edge of every other solar command pulse is used to generate the aspect sync signal. The generation of this signal is necessitated by the restrictions on the length of the data table and the read out rate controlled by the P1 program.

The centered sun pulse is also used to set the angular information into the shift register and to initiate an I.S.S. request. This is accomplished by setting the two code bits to their proper states and generating a data ready signal. The two code bits cause the subroutine to load the spacecraft clock into the data tables in two 12 bit words: (1) sun time fine and (2) sun time coarse. The subroutine then shifts the contents of the aspect system storage register into the data table. Hence, the occurrence of a centered sun pulse causes the sun time (event recorded by spacecraft clock) and the sun angle to be stored in three 12 bit words in the data table.

Earth Detector Electronics

The earth detector is a light sensing device which detects the gradient between the sunlit earth and outer space. The sensing element is a photo-diode having a response which peaks at one micron. As the satellite rotates, the detector's field of view sweeps out a cone. At certain points in the orbit, the scan of this detector successively crosses the horizons and/or terminator of the sunlit earth. When this occurs, the detector produces an electrical signal. This signal is fed into a high input impedance amplifier (earth amplifier module) whose output signal width corresponds to the apparent size of the sunlit earth.

Sun interference is removed from the earth signal by inhibiting the amplifier output when the sun's azimuth position is within ±45° of the earth detector's field of view. This is accomplished by counting the 16~/sp signal from the Data Sync. Clock in a four stage counter and decoding the contents to generate a pulse at the fourteenth and second counts. These decoded pulses clear and set a control flip-flop which generates an inhibit signal used to gate the output of the earth amplifier.

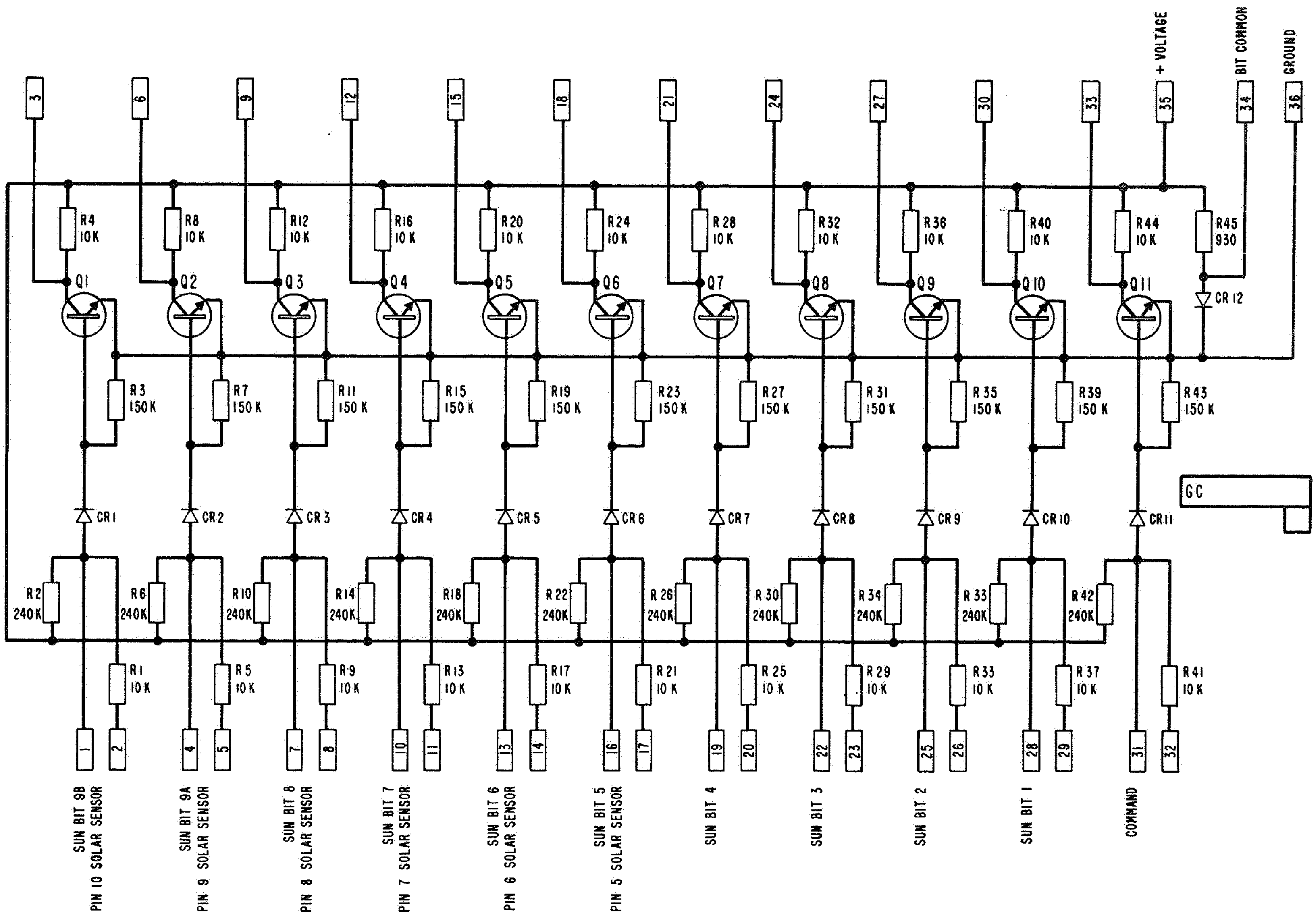

Figure 7. Solar Amplifier Schematic

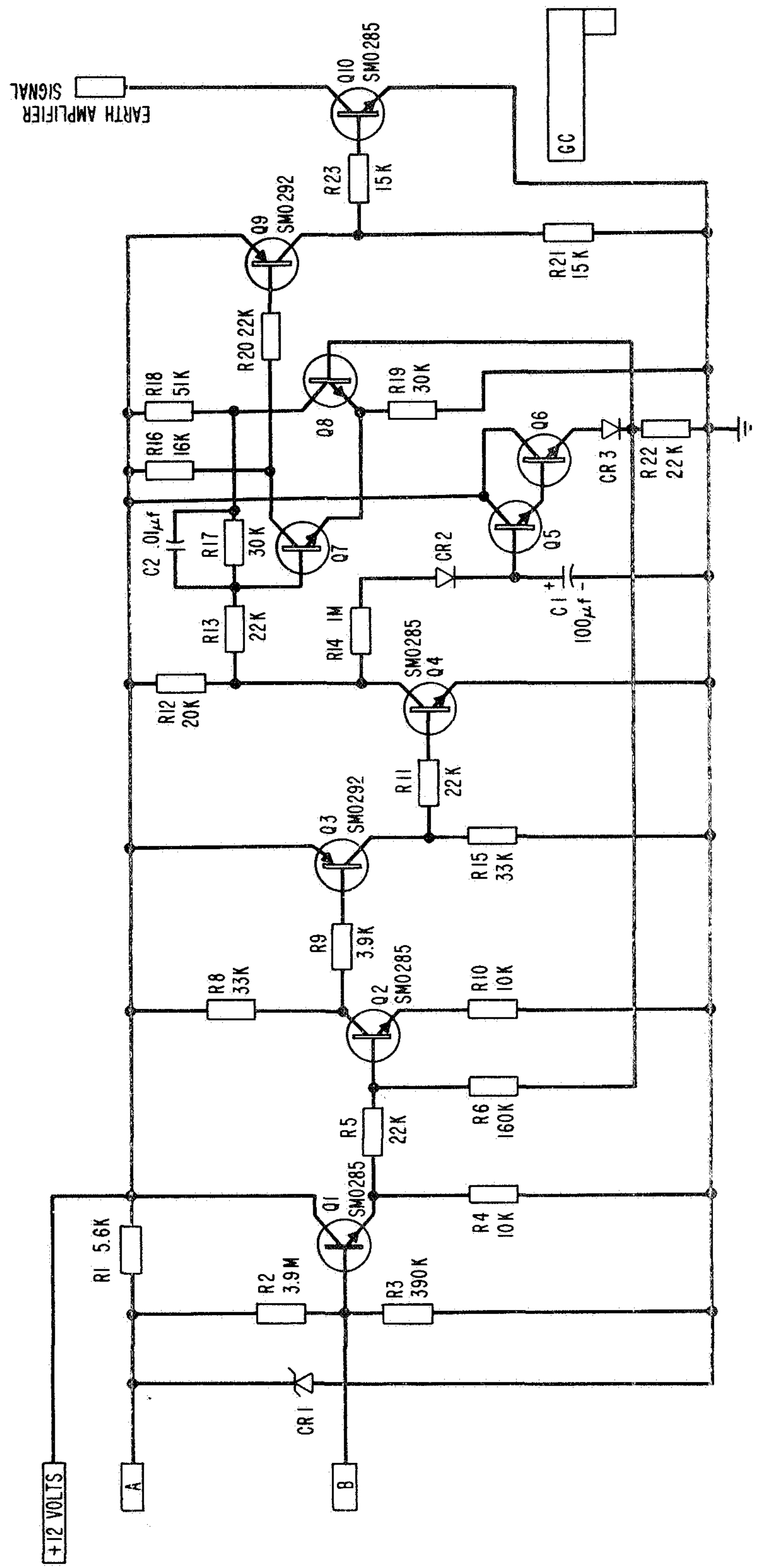

Figure 8. Earth Amplifier Schematic

The gated earth amplifier output is processed in such a manner that only the second earth signal following the aspect sync signal is allowed to generate leading and trailing edge pulses. The earth in (leading edge) and earth out (trailing edge) pulses both initiate separate I.S.S. requests. Each generate a data ready signal and the proper code bits to cause the subroutine to place both the coarse and fine spacecraft clock times into the data tables. Hence, an earth signal causes four twelve bit words to be stored in the data tables, two with the earth in pulse and two more with the earth out pulse. (See Figure 6).

Buffer Memory Failure Mode Electronics

The Aspect System provides for a buffer memory failure by supplying the storage and interval measuring logic necessary for operation with the telemetry format program (P1).

The aspect system is made aware of a buffer memory failure by the PMP1 logic control level ("1" indicates P1 control of aspect data). This logic control level nullifies the effect of the aspect sync. signal on the earth information, allowing every gated earth amplifier output to generate both leading (earth in) and trailing edge (earth out) pulses. These two earth pulses are then directed to a control circuit which alternately allow only one of the earth pulses through each spacecraft rotation period, i.e., in spin period N, the earth in pulse is allowed through, in spin period N+1, the earth out pulse is allowed through, etc. The output of this control circuit and the centered sun pulse are combined in an "OR" gate to form the event signal. Hence, in a single spin period where both the sunlit earth and the solar disc are detected, the event signal will consist of two pulses. One pulse will correspond to the sun crossing (centered sun pulse) and the second pulse to one of the two earth pulses. This event signal then resets two counters C_2 and C_{10}. The counter C_2 is a two stage device which inhibits its own input after two counts, and is used to generate a transfer pulse for the shift register. The counting source for C_2 is the telemetry frame signal and the transfer pulse is generated coincident with the second frame signal after an event occurred. The other counter C_{10} is a ten stage binary device which uses the spacecraft bit rate to measure the time interval between an event and the following telemetry frame signal. Hence, the interval between an event and a known time is measured by counter C_{10} and then transferred into the shift register with the second telemetry frame signal following the event. The occurrence of another aspect event before the contents of C_{10} have been transferred to the shift register nullifies the first event and the circuit logic proceeds to process the second event.

This type of operation necessitates the generation of inhibit signals to avoid the problem of loading the shift register before the data already present has been shifted to telemetry.

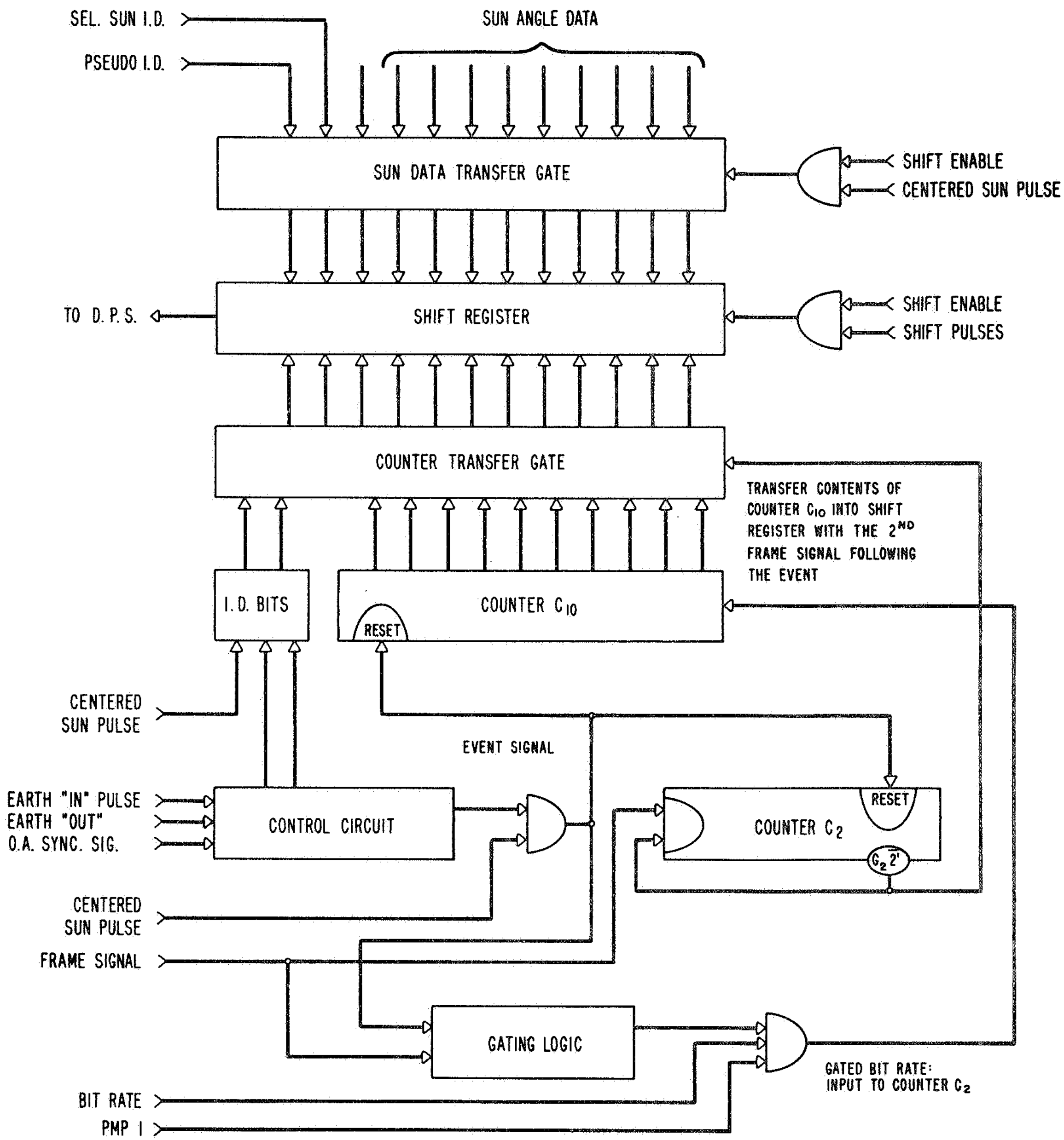

Figure 9. Aspect System Logic Diagram: Buffer Memory Failure Mode

	OPTICAL ASPECT WORD											
	BYTE N				BYTE N+1				BYTE N+2			
SUN ANGLE DATA	SUN BIT 1	SUN BIT 2	SUN BIT 3	SUN BIT 4	SUN BIT 5	SUN BIT 6	SUN BIT 7	SUN BIT 8	SUN BIT 9	0	SEL. SUN I.D.	PSEUDO-I.D.
SUN-FRAME END INTERVAL TIME	$\frac{2^0}{\text{BIT RATE}}$	$\frac{2^1}{\text{BIT RATE}}$	$\frac{2^2}{\text{BIT RATE}}$	$\frac{2^3}{\text{BIT RATE}}$	$\frac{2^4}{\text{BIT RATE}}$	$\frac{2^5}{\text{BIT RATE}}$	$\frac{2^6}{\text{BIT RATE}}$	$\frac{2^7}{\text{BIT RATE}}$	$\frac{2^8}{\text{BIT RATE}}$	$\frac{2^9}{\text{BIT RATE}}$	0	1
EARTH "IN"-FRAME END INTERVAL TIME	$\frac{2^0}{\text{BIT RATE}}$	$\frac{2^1}{\text{BIT RATE}}$	$\frac{2^2}{\text{BIT RATE}}$	$\frac{2^3}{\text{BIT RATE}}$	$\frac{2^4}{\text{BIT RATE}}$	$\frac{2^5}{\text{BIT RATE}}$	$\frac{2^6}{\text{BIT RATE}}$	$\frac{2^7}{\text{BIT RATE}}$	$\frac{2^8}{\text{BIT RATE}}$	$\frac{2^9}{\text{BIT RATE}}$	1	0
EARTH "OUT" FRAME END INTERVAL TIME	$\frac{2^0}{\text{BIT RATE}}$	$\frac{2^1}{\text{BIT RATE}}$	$\frac{2^2}{\text{BIT RATE}}$	$\frac{2^3}{\text{BIT RATE}}$	$\frac{2^4}{\text{BIT RATE}}$	$\frac{2^5}{\text{BIT RATE}}$	$\frac{2^6}{\text{BIT RATE}}$	$\frac{2^7}{\text{BIT RATE}}$	$\frac{2^8}{\text{BIT RATE}}$	$\frac{2^9}{\text{BIT RATE}}$	0	0

GC

Figure 10. Optical Aspect Word Format: P1 Only

One type of inhibit is accomplished by waiting until the second telemetry frame signal following an event before transferring data from the counter C_{10} to the shift register. Since the data processing system (D.P.S.) supplies the aspect system with an effective burst of shift pulses every frame, the register is read out once per frame. Hence, even if the event occurred after the shift pulses in a specific frame, the register will be cleared in the next frame allowing the aspect event data to be transferred without loading over existing data.

A second inhibit function is needed when considering the occurrence of a centered sun pulse. As previously mentioned, the centered sun pulse transfers the solar elevation data into the shift register as well as initiating the measurement of a time interval. Therefore, if the shift register is transferring information to telemetry under control of P1 when a centered sun pulse occurs, the loading of solar elevation data must be inhibited but the initiation of an event measurement can still be allowed. This is accomplished by using the shift enable signal to lock out the load pulse generated by the centered sun pulse.

The aspect data generated in this mode must be identified. This is accomplished by adding two bits of information to the contents of the shift register. Two flip-flops are used for these identification bits and their presence in the aspect word defines the aspect event associated with the data, i.e., earth in, earth out, or centered sun pulse.

Evaluation of Aspect Parameters

The quantities measured by the Optical Aspect System are:

1. Spin axis-sun angle
2. Sun time
3. Earth "in" time
4. Earth "out" time

As previously mentioned, the three parameters necessary for attitude determination are: (1) β, the spin-axis sun angle, (2) δ, the spin axis earth angle, (3) the fraction of a spin period between observing each.

The spin axis-sun angle (β) is measured directly by the solar sensor and its digital representation placed in storage. To convert the digital representation of the gray-coded sun data to a numerical value of β, the nine bits must first be inverted. Sun bit 9 represents the sensor reticle identification.

$0 < \beta \leq 90°$	sun bit 9 = "0" level
$90° < \beta \leq 180°$	sun bit 9 = "1" level

OPTICAL ASPECT WORD												
BYTE N				BYTE N+1				BYTE N+2				
1	2	3	4	1	2	3	4	1	2	3	4	
X	X	X	X	X	X	X	X	0	0	0	0	EARTH OUT-FRAME END TIME:
X	X	X	X	X	X	X	X	0	0	0	0	SUN PULSE-FRAME END TIME:
X	X	X	X	X	X	X	X	0	0	1	0	EARTH IN-FRAME END TIME:
X	X	X	X	X	X	X	X	0	0	1	1	ERROR
X	X	X	X	X	X	X	X	0	1	0	0	EARTH OUT-FRAME END TIME:
X	X	X	X	X	X	X	X	0	1	0	1	SUN PULSE-FRAME END TIME:
X	X	X	X	X	X	X	X	0	1	1	0	EARTH IN-FRAME END TIME:
X	X	X	X	X	X	X	X	0	1	1	1	ERROR
X	X	X	X	X	X	X	X	1	0	0	0	EARTH OUT-FRAME END TIME:
X	X	X	X	X	X	X	X	1	0	0	1	SUN PULSE-FRAME END TIME-OR-SUN ANGLE DATA:
X	X	X	X	X	X	X	X	1	0	1	0	EARTH IN-FRAME END TIME:
X	X	X	X	X	X	X	X	1	0	1	1	SUN ANGLE DATA
X	X	X	X	X	X	X	X	1	1	0	0	EARTH OUT-FRAME END TIME:
X	X	X	X	X	X	X	X	1	1	0	1	SUN PULSE-FRAME END TIME:
X	X	X	X	X	X	X	X	1	1	1	0	EARTH IN-FRAME END TIME:
X	X	X	X	X	X	X	X	1	1	1	1	NO DATA

GC

Figure 11. Optical Aspect Word I.D.: P1 Only

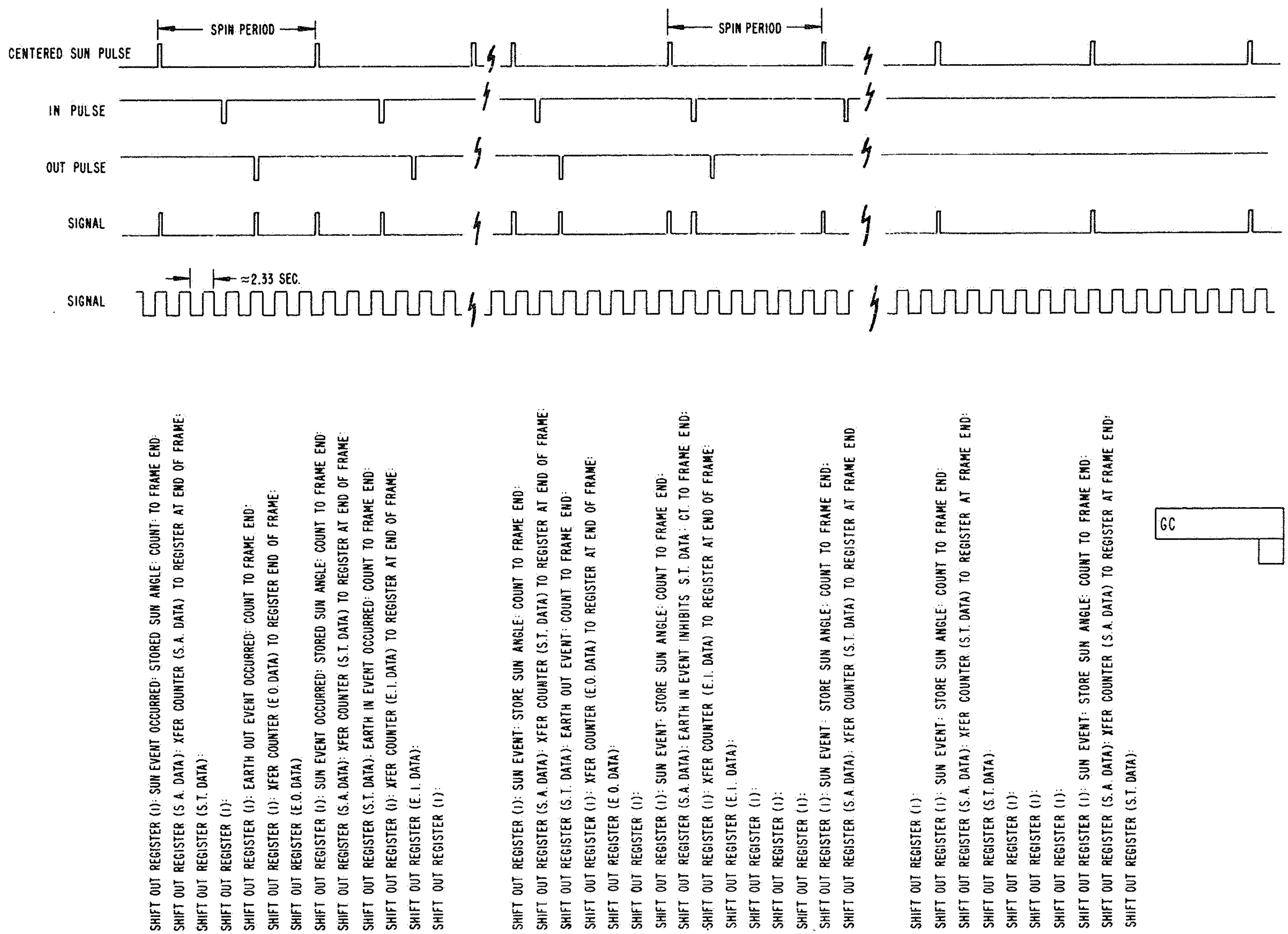

Figure 12. Optical Aspect Word Sequence: P1 Only

The remaining 8 bits of sun data is converted to a decimal equivalent in the following manner:

A. Gray-code to Binary

1) the most significant bit (8th) is the same for both codes
2) for all the following bits

a) if the preceding binary bit is a "1", the gray coded bit is inverted to form the next binary bit.
b) if the preceding binary bit is a "0", the gray coded bit is the same as the next binary bit.

B. Binary-to-Decimal

1) conversion from binary to decimal is accomplished by weighting each bit by the appropriate power of two and adding the results to obtain N_T

8th sun bit - 2^6

7th sun bit - 2^5

6th sun bit - 2^4

5th sun bit - 2^3

4th sun bit - 2^2

3rd sun bit - 2^1

2nd sun bit - 2^0

1st sun bit - 2^{-1}

C. $N_S = N_T + 0.25$

D. ($90° - N_S$ if sun bit 9 = "0")

($90° + N_S$ if sun bit 9 = "1")

When the buffer memory is used to process the aspect data, the time occurrence of aspect events are recorded by the spacecraft clock. Hence, sun time, earth "in" time and earth "out" time are available from the telemetry output.

These times are then related to G. M. T. by use of the frame count and sub-com. time information. This is not the case when the D. P. S. is operating on the aspect data through the use of P1, i.e.: buffer memory failure mode. The events are measured with respect to the end of a telemetry frame and read out in the second telemetry frame following the event. Hence, the time interval data associated with an event that occurred during telemetry frame N will appear as output data in telemetry frame N+2. To obtain the time that the event occurred, the aspect word value plus one frame time must be subtracted from the time value of the frame count in the frame in which the aspect data was read out. Therefore, in either mode of operation, the value of the aspect parameter's i.e.; sun time; earth "in" time, earth "out" time, can be obtained from the telemetry data.

Now, examine the following definitions;

a) Spin Period = time interval between two successive sun occurrences.
b) Earth time = time interval between the occurrence of a sun pulse and an earth "in" pulse.
c) Earth Width = time interval between an earth "in" pulse and an earth "out" pulse.

Hence, from the time occurrence values of the aspect events, the three parameters defined above can be found. Then, the fraction of a spin period between observing the two reference sources (sun and earth) can be determined from these same three parameters.

APPENDIX A

MATHEMATICAL REPRESENTATION OF EULERIAN ROTATIONS

Any vector $(\bar{N})$ in the (X, Y, Z) coordinate system, written in column form, may be transformed into the (ξ', η', ζ') system by the application of the rotation matrix D.

$$\begin{vmatrix} N_{\xi'} \\ N_{\eta'} \\ N_{\zeta'} \end{vmatrix} = D \begin{vmatrix} N_X \\ N_Y \\ N_Z \end{vmatrix} = \begin{vmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} N_X \\ N_Y \\ N_Z \end{vmatrix}$$

This vector, when acted upon by the rotation matrix C yields:

$$\begin{vmatrix} N_{\xi} \\ N_{\eta} \\ N_{\zeta} \end{vmatrix} = CD \begin{vmatrix} N_X \\ N_Y \\ N_Z \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{vmatrix} \begin{vmatrix} \cos\phi & \cos\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} N_X \\ N_Y \\ N_Z \end{vmatrix}$$

Finally, application of the rotation matrix B produces the N vector transformed to the (x, y, z) coordinate system.

$$\begin{vmatrix} N_x \\ N_y \\ N_z \end{vmatrix} = BCD \begin{vmatrix} N_X \\ N_Y \\ N_Z \end{vmatrix} = \begin{vmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{vmatrix} \begin{vmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} N_X \\ N_Y \\ N_Z \end{vmatrix}$$

Hence, any vector in the (X, Y, Z) coordinate system can be determined in the (x, y, z) coordinate system by the application of the transformation matrix A = BCD. Performing the matrix multiplication:

$$A = \begin{vmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi & \sin\psi\sin\theta \\ -\sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi & -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi & \cos\psi\sin\theta \\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta \end{vmatrix}$$

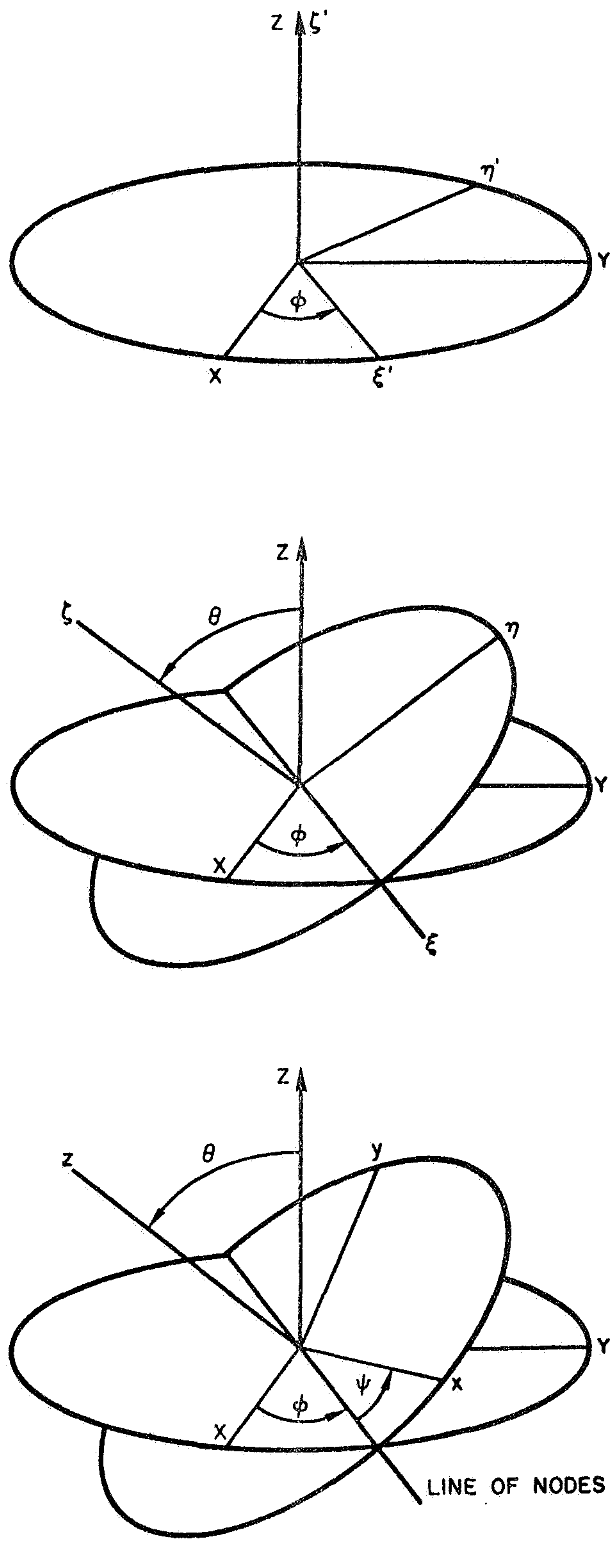

Figure 13. Rotations Defining the Eulerian Angles

Since A is an orthogonal transformation: $A^{-1} = A^T$ and any vector in the (x, y, z) coordinate system can be transformed to the (X, Y, Z) system by application of the transformation matrix A^{-1}

$$A^{-1} = \begin{vmatrix} \cos\phi\cos\psi - \sin\phi\cos\theta\sin\psi & -\sin\psi\cos\phi - \cos\theta\cos\psi\sin\phi & \sin\theta\sin\phi \\ \sin\phi\cos\psi + \cos\theta\cos\phi\sin\psi & -\sin\psi\sin\phi + \cos\theta\cos\psi\cos\phi & -\sin\theta\cos\phi \\ \sin\theta\sin\psi & \sin\theta\cos\psi & \cos\theta \end{vmatrix}$$

APPENDIX B

MOMENTUM AND ANGULAR VELOCITIES

A. MOMENTUM

Consider the momentum vector ($\bar{L}$) of a rotating satellite orientated along the Z-axis of the XYZ coordinate system. Writing the momentum vector as a column vector in the XYZ system we have

$$\bar{L} = \begin{vmatrix} L_X \\ L_Y \\ L_Z \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ L \end{vmatrix}$$

Hence, transforming the momentum vector to the xyz coordinate system, the instantaneous values of momentum about the xy and z axes are obtained. Denoting these values as p_x, p_y, and p_z respectively we have

$$\begin{vmatrix} p_x \\ p_y \\ p_z \end{vmatrix} = AL = A\begin{vmatrix} 0 \\ 0 \\ L \end{vmatrix} = \begin{vmatrix} L \sin\psi \sin\theta \\ L \cos\psi \sin\theta \\ L \cos\theta \end{vmatrix}$$

and

$$p_x = L \sin\psi \sin\theta$$

$$p_y = L \cos\psi \sin\theta$$

$$p_z = L \cos\theta$$

B. ANGULAR VELOCITIES

Consider $\omega_\phi = d\phi/dt = \dot{\phi}$ as vector directed along the Z-axis. Writing ϕ as a column vector

$$\dot{\phi} = \begin{vmatrix} 0 \\ 0 \\ \dot{\phi} \end{vmatrix}$$

Then expressing $\dot{\phi}$ in the xyz coordinate system we have:

$$\begin{vmatrix} \omega_{\phi_x} \\ \omega_{\phi_y} \\ \omega_{\phi_z} \end{vmatrix} = \begin{vmatrix} \dot{\phi}_x \\ \dot{\phi}_y \\ \dot{\phi}_z \end{vmatrix} = A\dot{\phi} = A\begin{vmatrix} 0 \\ 0 \\ \dot{\phi} \end{vmatrix} = \begin{vmatrix} \dot{\phi}\sin\psi\sin\theta \\ \dot{\phi}\cos\psi\sin\theta \\ \dot{\phi}\cos\theta \end{vmatrix}$$

and

$$\omega_{\phi_x} = \dot{\phi}\sin\psi\sin\theta$$

$$\omega_{\phi_y} = \dot{\phi}\cos\psi\sin\theta$$

$$\omega_{\phi_z} = \dot{\phi}\cos\theta$$

Next consider $\omega_\theta = d\theta/dt = \dot{\theta}$ as a vector directed along the line of nodes. Writing $\dot{\theta}$ as a column vector in the XYZ system

$$\dot{\theta} = \begin{vmatrix} \dot{\theta}\cos\phi \\ \dot{\theta}\sin\phi \\ 0 \end{vmatrix}$$

and transforming this vector to the xyz system

$$\begin{vmatrix} \omega_{\theta_x} \\ \omega_{\theta_y} \\ \omega_{\theta_z} \end{vmatrix} = \begin{vmatrix} \theta_x \\ \theta_y \\ \theta_z \end{vmatrix} = A\dot{\theta} = A\begin{vmatrix} \dot{\theta}\cos\phi \\ \dot{\theta}\sin\phi \\ 0 \end{vmatrix} = \begin{vmatrix} \dot{\theta}\cos\psi \\ -\dot{\theta}\sin\psi \\ 0 \end{vmatrix}$$

$$\omega_{\theta_x} = \dot{\theta}\cos\psi$$

$$\omega_{\theta_y} = -\dot{\theta}\sin\psi$$

$$\omega_{\theta_z} = 0$$

Finally, consider $\omega_\psi = d\psi/dt = \dot{\psi}$ a vector perpendicular to the plane containing the line of nodes and the x-axis and directed along the z-axis. Hence

$$\begin{vmatrix} \omega_{\psi_x} \\ \omega_{\psi_y} \\ \omega_{\psi_z} \end{vmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{vmatrix} 0 \\ 0 \\ \dot{\psi} \end{vmatrix}$$

Now combining the x, y, and z components of $\dot{\phi}$, $\dot{\psi}$ and $\dot{\theta}$, a relationship is established between the angular velocities ω_x, ω_y, and ω_z and the rate of change of the Euler angles.

$$\omega_x = \dot{\phi} \sin\psi \sin\theta + \dot{\theta} \cos\psi$$

$$\omega_y = \dot{\phi} \cos\psi \sin\theta - \dot{\theta} \sin\psi$$

$$\omega_z = \dot{\phi} \cos\theta + \dot{\psi}$$

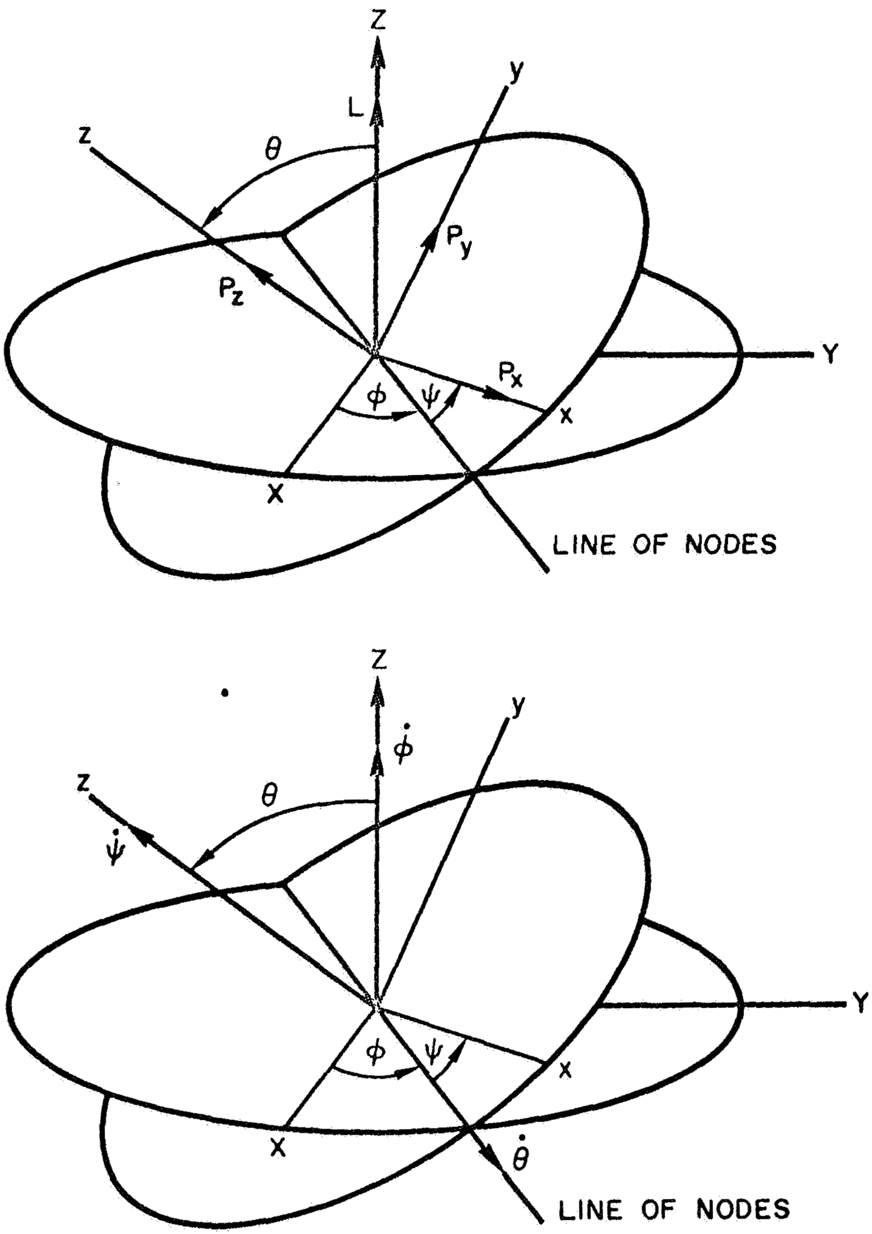

Figure 14. Components of Angular Momentum and Velocities

APPENDIX C

CALCULATION OF DELTA (δ)

Referring to Figure 15b and considering the plane containing $\overline{EA}$ and $\overline{EC}$ we have that

$$\cos \frac{\pi \Delta t_a}{T_E} = \frac{\overline{EB}}{\overline{EA}} \tag{3-1}$$

where Δt_a is the time between horizon pulses and T_E is the rotation period. From Figure 14, various other relationships can be derived:

$$\overline{EB} = \overline{ED} + \overline{DB} \tag{3-2}$$

$$\overline{ED} = \overline{OE} \tan(90 - \delta) \quad \text{where} \quad \overline{OE} = \overline{OH} \cos\gamma \tag{3-3}$$

$$\overline{DB} = \frac{\overline{OG}}{\cos(90 - \delta)} \quad \text{where} \quad \overline{OG} = \overline{OH} \cos\rho \tag{3-4}$$

$$\overline{EA} = \overline{OH} \sin\gamma \tag{3-5}$$

Now considering Equation (3-1)

$$\frac{\overline{EB}}{\overline{EA}} = \cos \frac{\pi \Delta t_a}{T_E}$$

But from Equations (3-2) and (3-3)

$$\overline{EB} = \overline{ED} + \overline{DB}$$

$$\overline{ED} = \overline{OE} \tan(90 - \delta)$$

Therefore:

$$\overline{ED} = \overline{OH} \cos\gamma \tan(90 - \delta)$$

From Equation (3-4)

$$\overline{DB} = \frac{\overline{OH} \cos\rho}{\cos(90 - \delta)}$$

hence

$$\overline{EB} = \overline{OH} \cos\gamma \tan(90 - \delta) + \frac{\overline{OH} \cos\rho}{\cos(90 - \delta)}$$

or since

$$\tan(90 - \delta) = \cot\delta$$

and

$$\cos(90 - \delta) = \sin\delta$$

$$\overline{EB} = \overline{OH} \cos\gamma \cot\delta + \frac{\overline{OH} \cos\rho}{\sin\delta}$$

Finally from Equation (3-5)

$$\overline{EA} = \overline{OH} \sin\gamma$$

Then:

$$\overline{OH}\left[\cos\gamma \cot\delta + \frac{\cos\rho}{\sin\delta}\right] = \overline{EA} \cos\frac{\pi\Delta t_a}{T_E} \tag{3-6}$$

$$\overline{OH}\left[\cos\gamma\cot\delta + \frac{\cos\rho}{\sin\delta}\right] = \overline{OH}\sin\gamma\cos\frac{\pi\Delta t_a}{T_E} \tag{3-7}$$

$$\sin\delta\cos\gamma\cot\delta + \cos\rho = \sin\delta\sin\gamma\cos\frac{\pi\Delta t_a}{T_E} \tag{3-8}$$

$$\cos\gamma\cos\delta + \cos\rho = \sin\delta\sin\gamma\cos\frac{\pi\Delta t_a}{T_E} \tag{3-9}$$

Let

$$d = \cos\rho$$

$$\epsilon = \cos\gamma$$

$$f = \sin\gamma\cos\frac{\pi\Delta t_a}{T_E}$$

Then

$$d + \epsilon\cos\delta = f\sin\delta = f\sqrt{1-\cos^2\delta} \tag{3-10}$$

Now squaring both sides

$$d^2 + 2\epsilon d\cos\delta + \epsilon^2\cos^2\delta + f^2\cos^2\delta - f^2 = 0$$

or

$$\left(\epsilon^2 + f^2\right)\cos^2\delta + 2\epsilon d\cos\delta + \left(d^2 - f^2\right) = 0 \tag{3-11}$$

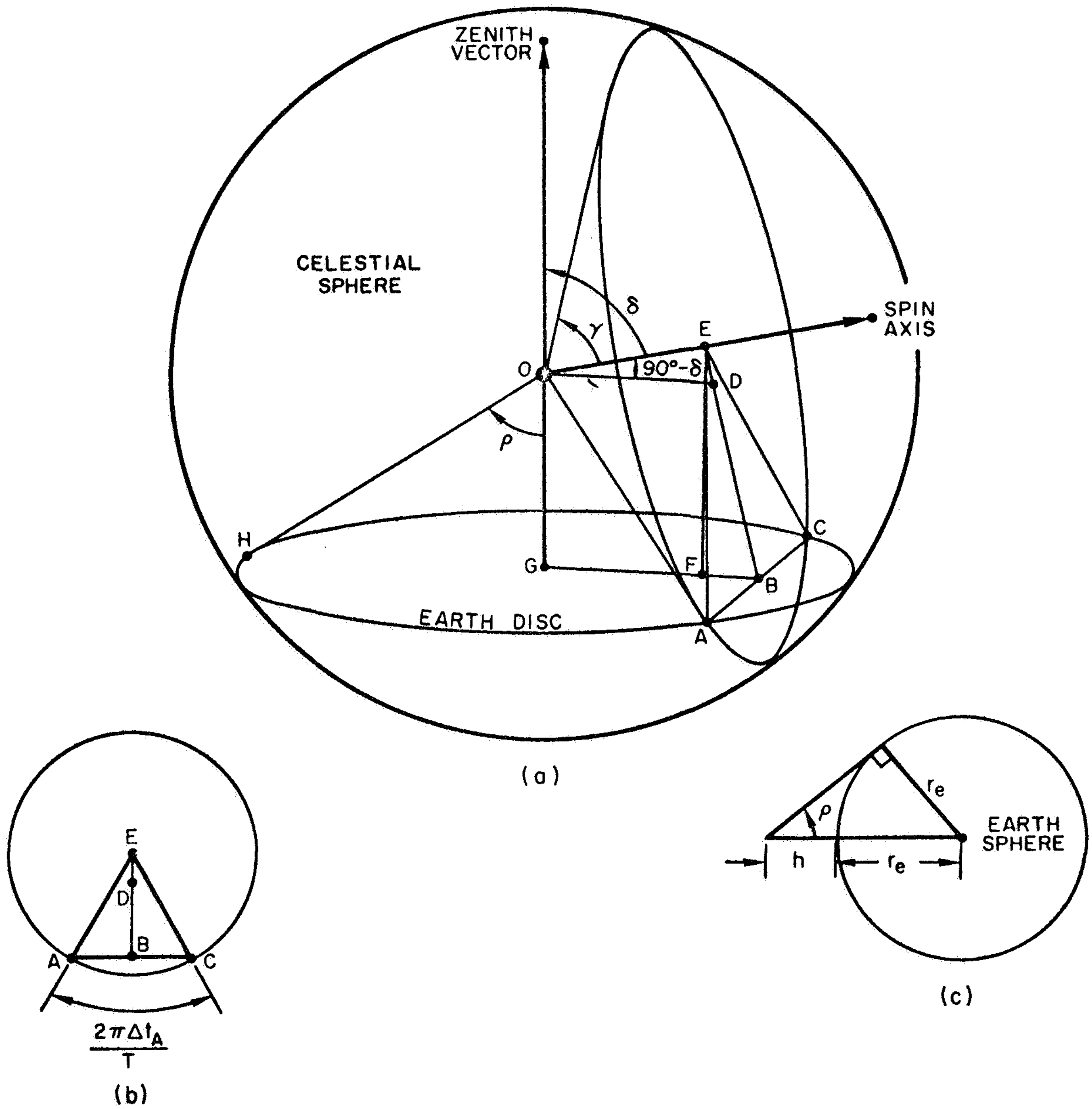

Figure 15. Definition of Variables for the Two Horizon Solution for Delta

using the quadradic expression to solve for

$$\cos\delta = \frac{-\epsilon\, d \pm \sqrt{d^2\,\epsilon^2 - \left(\epsilon^2 + f^2\right)\left(d^2 - f^2\right)}}{e^2 + f^2}$$

or

$$\cos\delta = \frac{-d\epsilon \pm f\sqrt{\epsilon^2 + f^2 - d^2}}{e^2 + f^2}$$

Figure 15 represents a satellite rotating with no precession. An earth horizon sensor is mounted $\gamma°$ off the satellite z-axis so as to sweep out a cone which cuts the earth horizons at A and C. The inclination (δ) of the momentum vector to the sub-satellite zenith vector may be expressed in terms of γ, the satellite elevation and the horizon pulse spacing.

$$\cos\delta = \frac{-de \pm f\sqrt{\epsilon^2 + f^2 - d^2}}{\epsilon^2 + f^2} \qquad 0 \le \delta \le 180° \tag{23}$$

where

$$d = \cos\rho$$

$$\epsilon = \cos\gamma$$

$$f = \sin\gamma\cos\left(\frac{\pi\Delta\, t_a}{T_E}\right)$$

and

$$\rho = \sin^{-1}\left(\frac{r_E}{r_E + h}\right)$$

Δt_a = horizon pulse spacing

T_E = rotation period.

Information determining the value of δ is also contained in the relative position of the horizon pulses to the command slit crossing of the sun. On the satellite the earth horizon sensor has a pencil field of view located $\gamma°$ from the satellite z-axis in the plane of the command slit. If the satellite is spinning so that $\theta = 0$, horizon pulses will be symmetrical about the time at which this plane crosses the center of the earth (see Figure 15). The center of the earth is located on the celestial sphere at (RA_z + 180°) and ($-D_z$), where RA_z is the right ascension and D_z is the declination of the subsatellite point zenith vector. (180° - η) is the great circle arc from the sun to the earth's center on the celestial sphere. Δt_c is the time midway between horizon pulses minus the time the command slit crosses the sun. The relationship between δ and Δt_c can now be written from the spherical triangles in Figure 16.

$$\cos(180° - \eta) = \cos\beta\cos(180 - \delta) + \sin\beta\sin(180 - \delta)\cos\left[(\dot{\phi} + \dot{\psi})\Delta t_c\right] \quad (24)$$

$$\cos\eta = \cos\beta\cos\delta - \sin\beta\sin\delta\cos\left[(\dot{\phi} + \dot{\psi})\Delta t_c\right] \quad (25)$$

Solving for δ gives:

$$\cos\delta = \frac{rs \pm \mu\sqrt{s^2 + \mu^2 - r^2}}{s^2 + \mu^2} \qquad 0 \leq \delta \leq \pi \quad (26)$$

where

$$r = -\cos\eta$$

$$s = -\cos\beta$$

$$\mu = \sin\beta\cos\left[(\dot{\phi} + \dot{\psi})\Delta t_c\right]$$

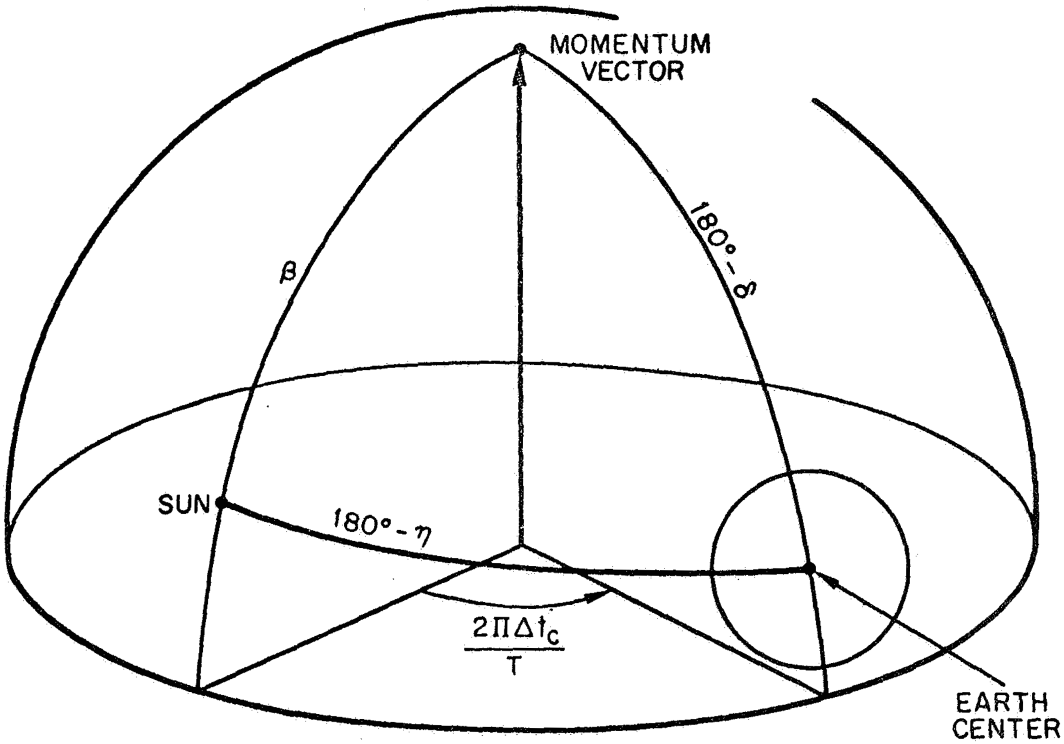

Figure 16. Relationship Between Δt_c and Delta

and where

$$\cos \eta = \sin D_z \sin D_s + \cos D_z \cos D_s \cos (RA_s - RA_z) \qquad 0 \leq \eta \leq 180° \quad (27)$$

D_z = declination of subsatellite zenith

D_s = declination of sun

RA_z = right ascension of subsatellite zenith

RA_s = right ascension of sun.

APPENDIX D

SCHEMATIC DIAGRAMS

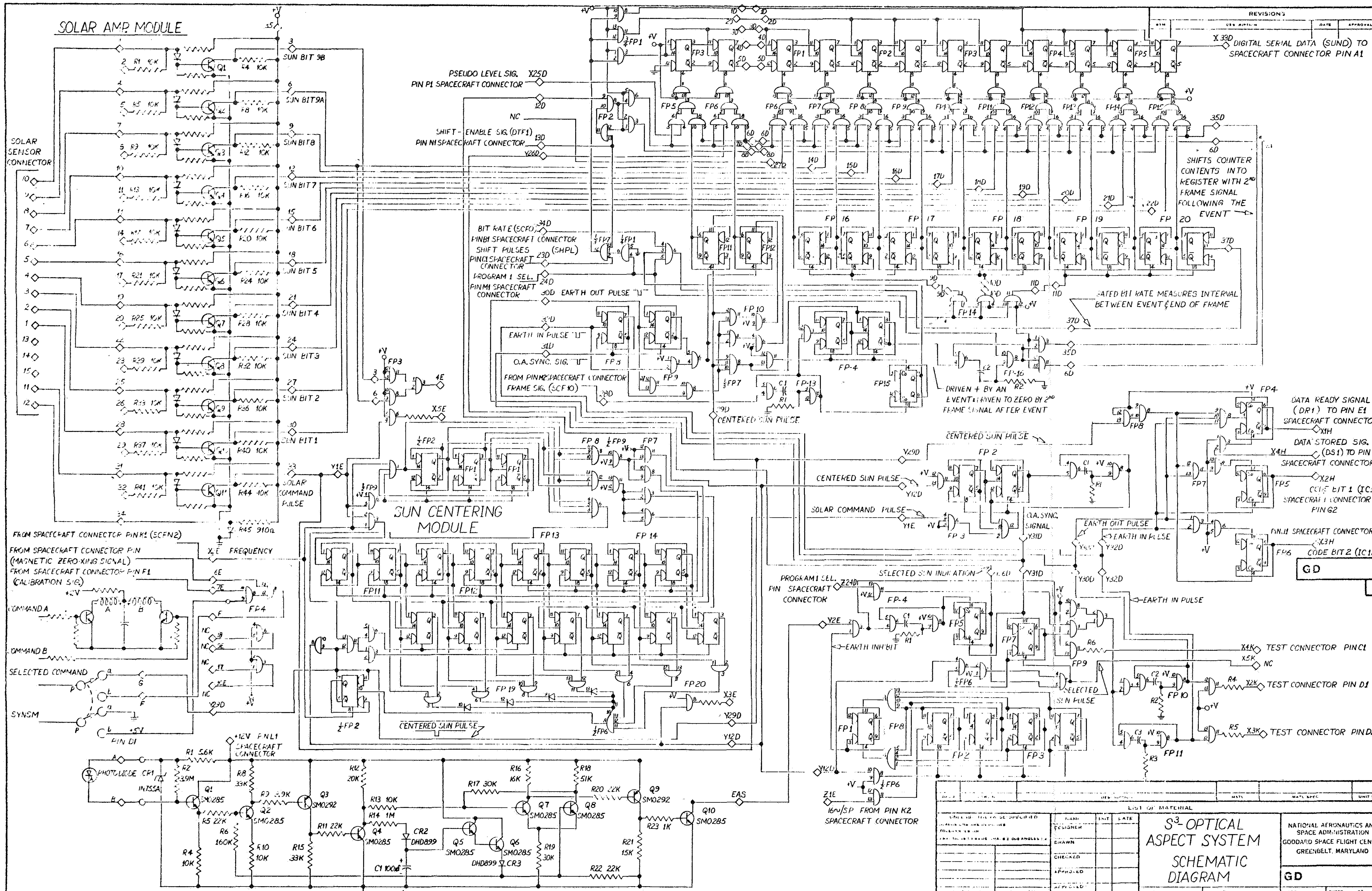

Figure 17. Optical Aspect System: Schematic Diagram

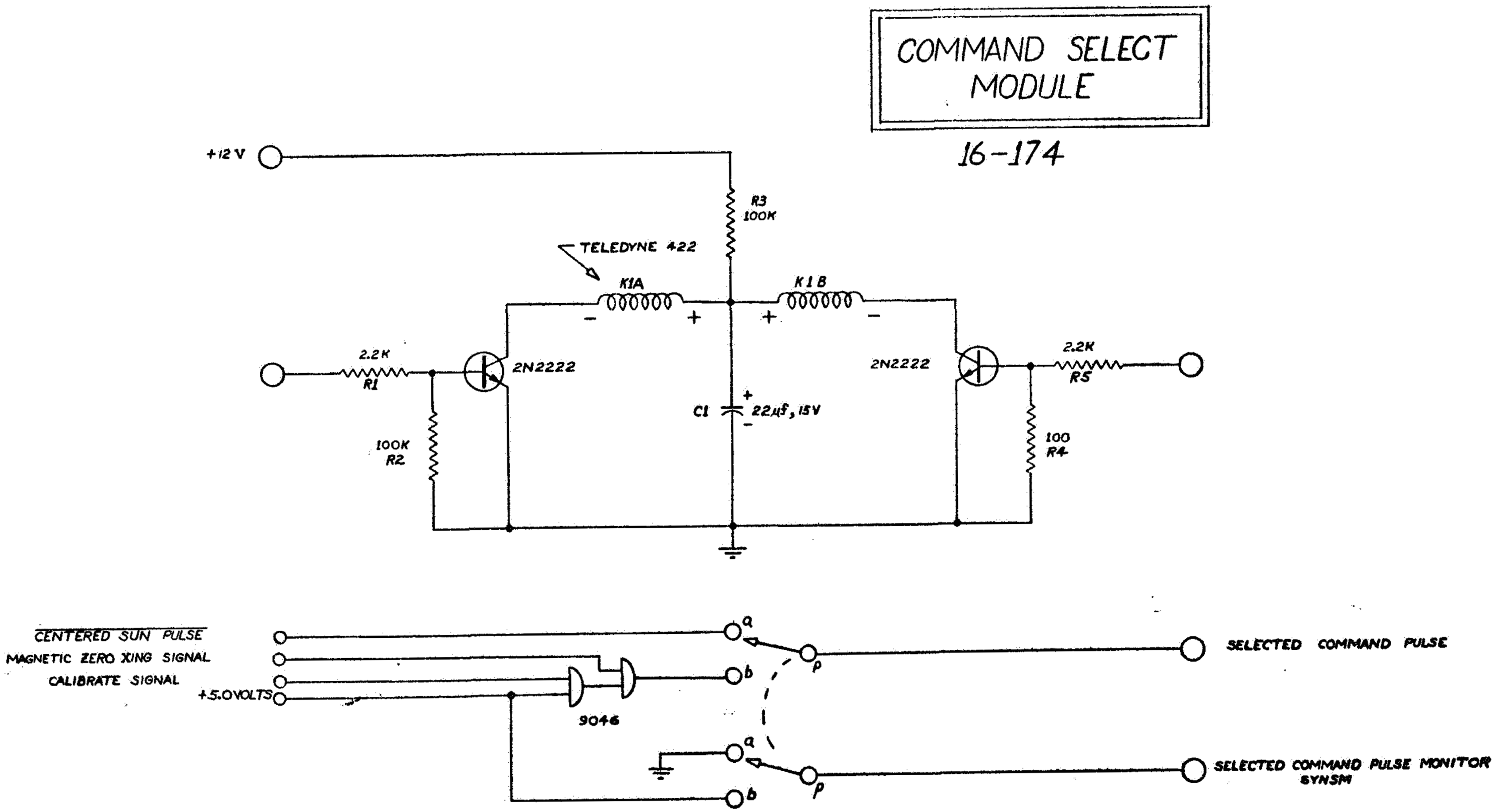

Figure 18. Command Select Module

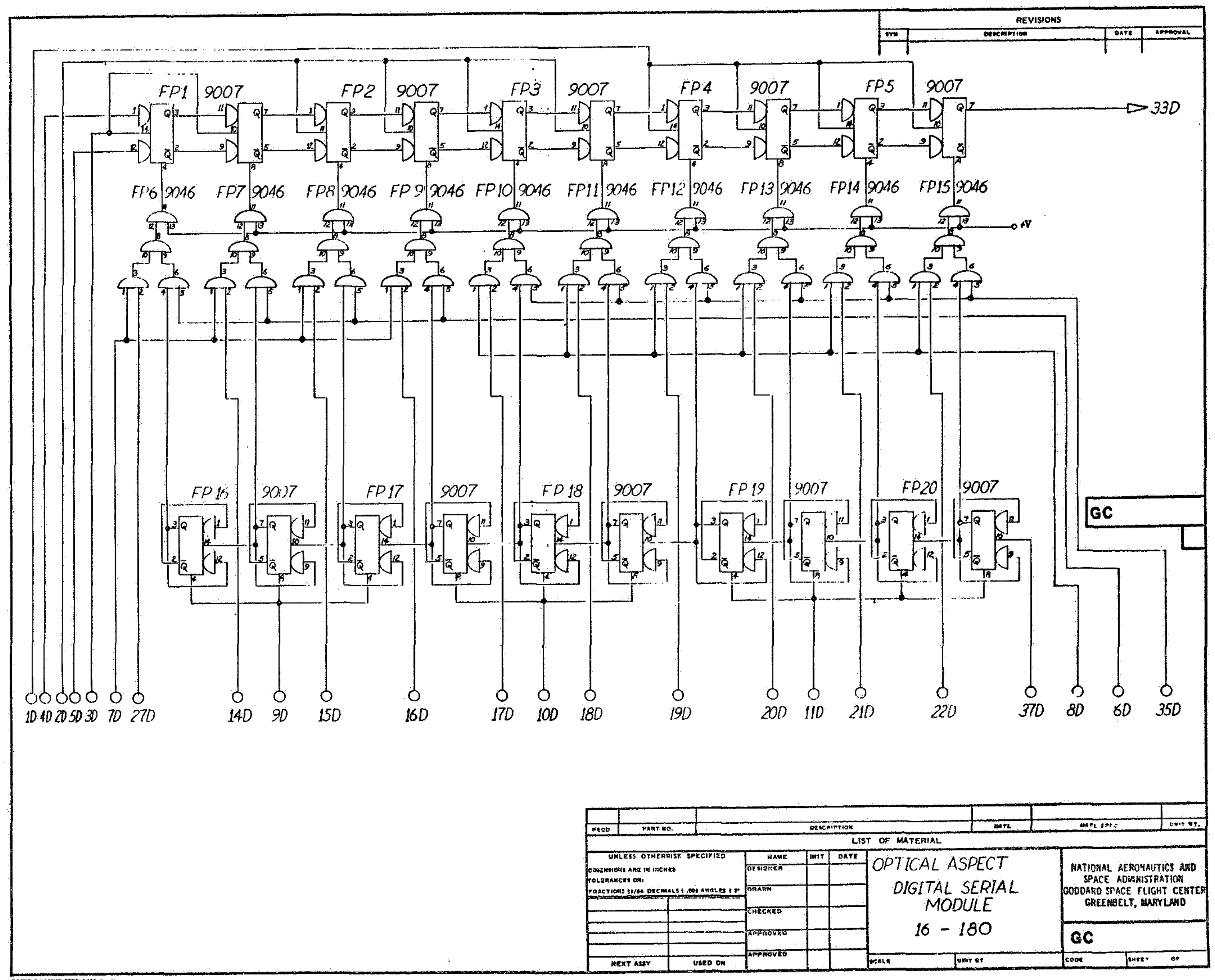

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Figure 19. Optical Aspect Digital Serial Module 16-180

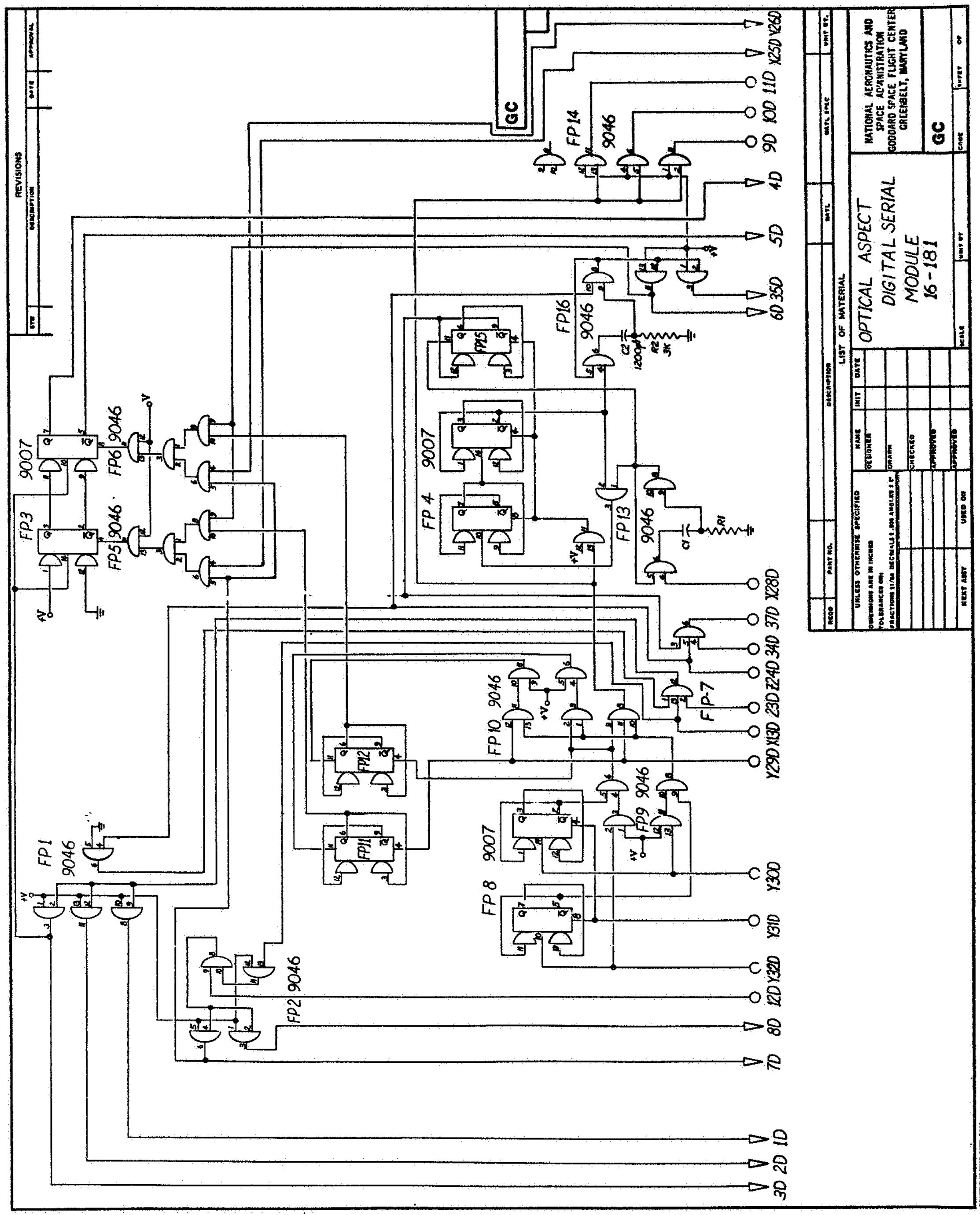

Figure 20. Optical Aspect Digital Serial Module 16-181

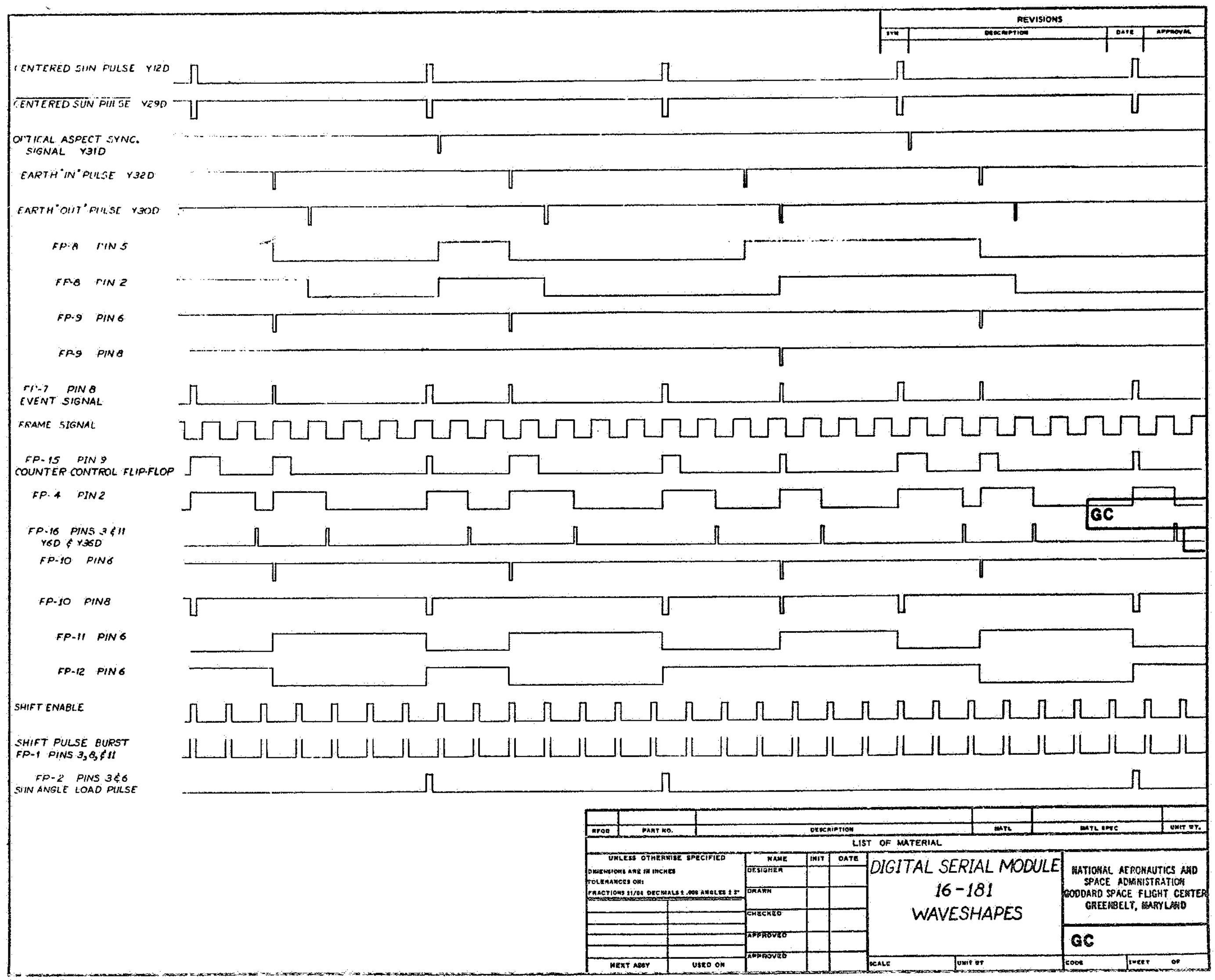

Figure 21. Digital Serial Module 16-181 Waveshapes

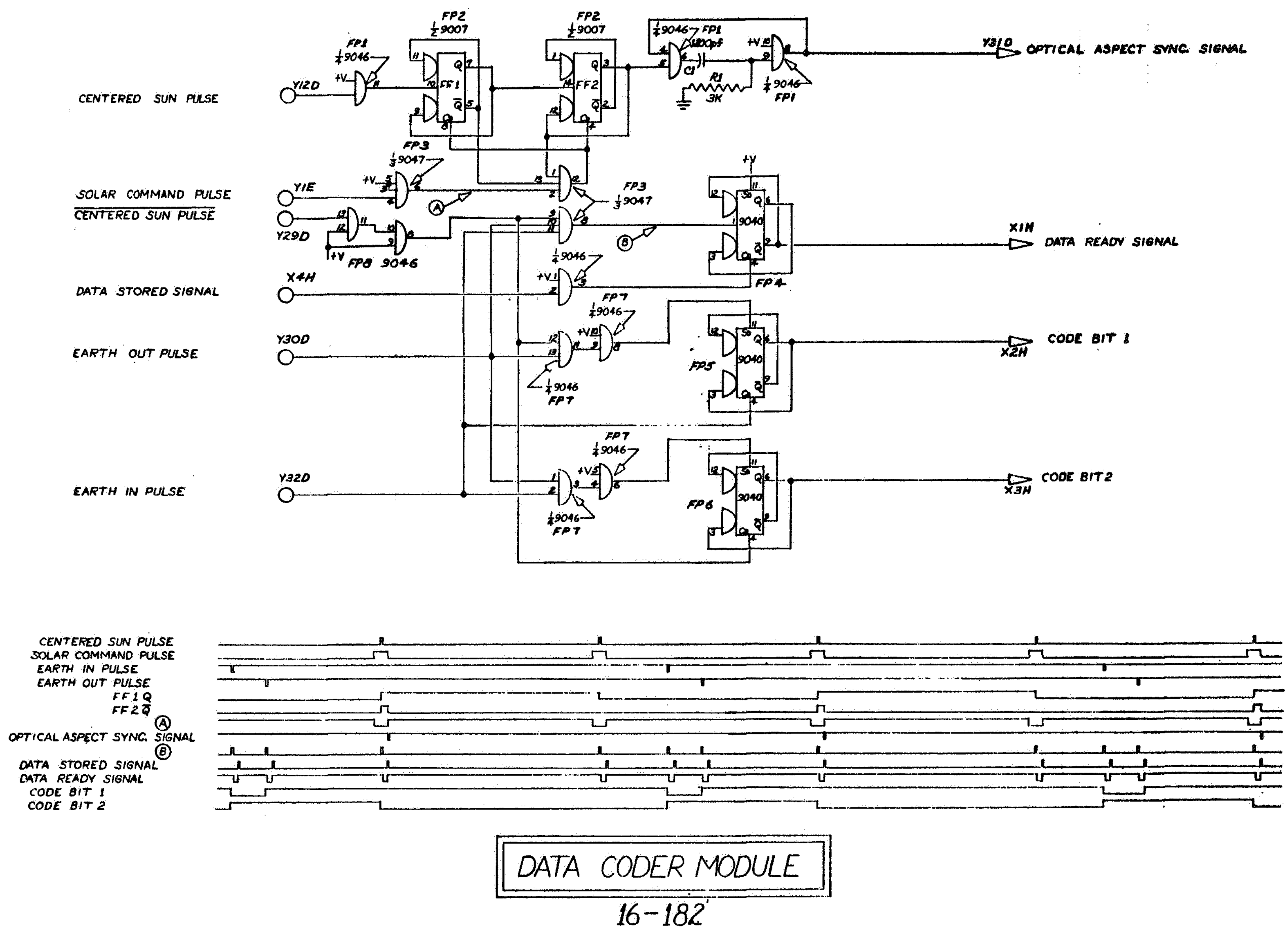

Figure 22. Data Coder Module

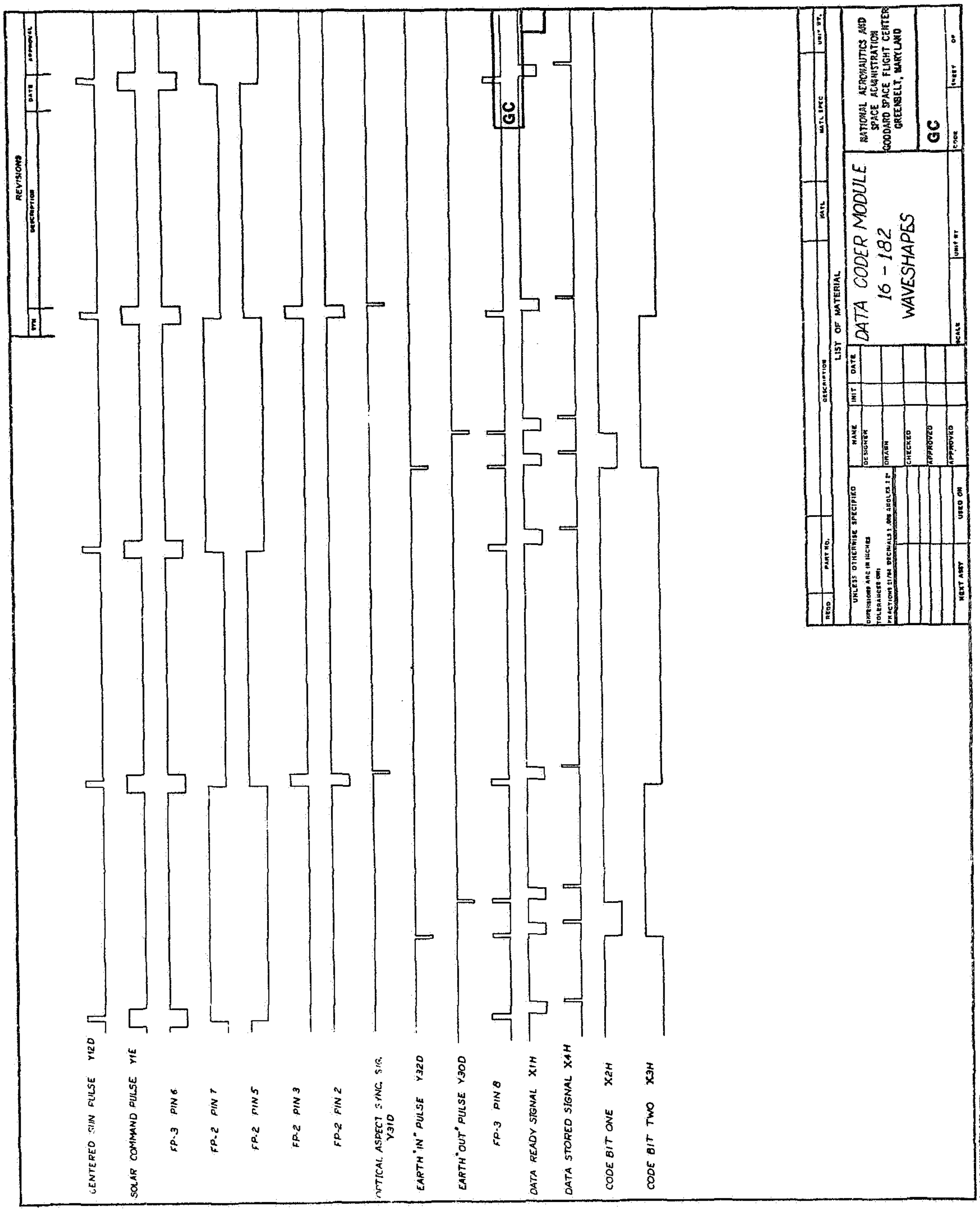

Figure 23. Data Coder Module 16-182 Waveshapes

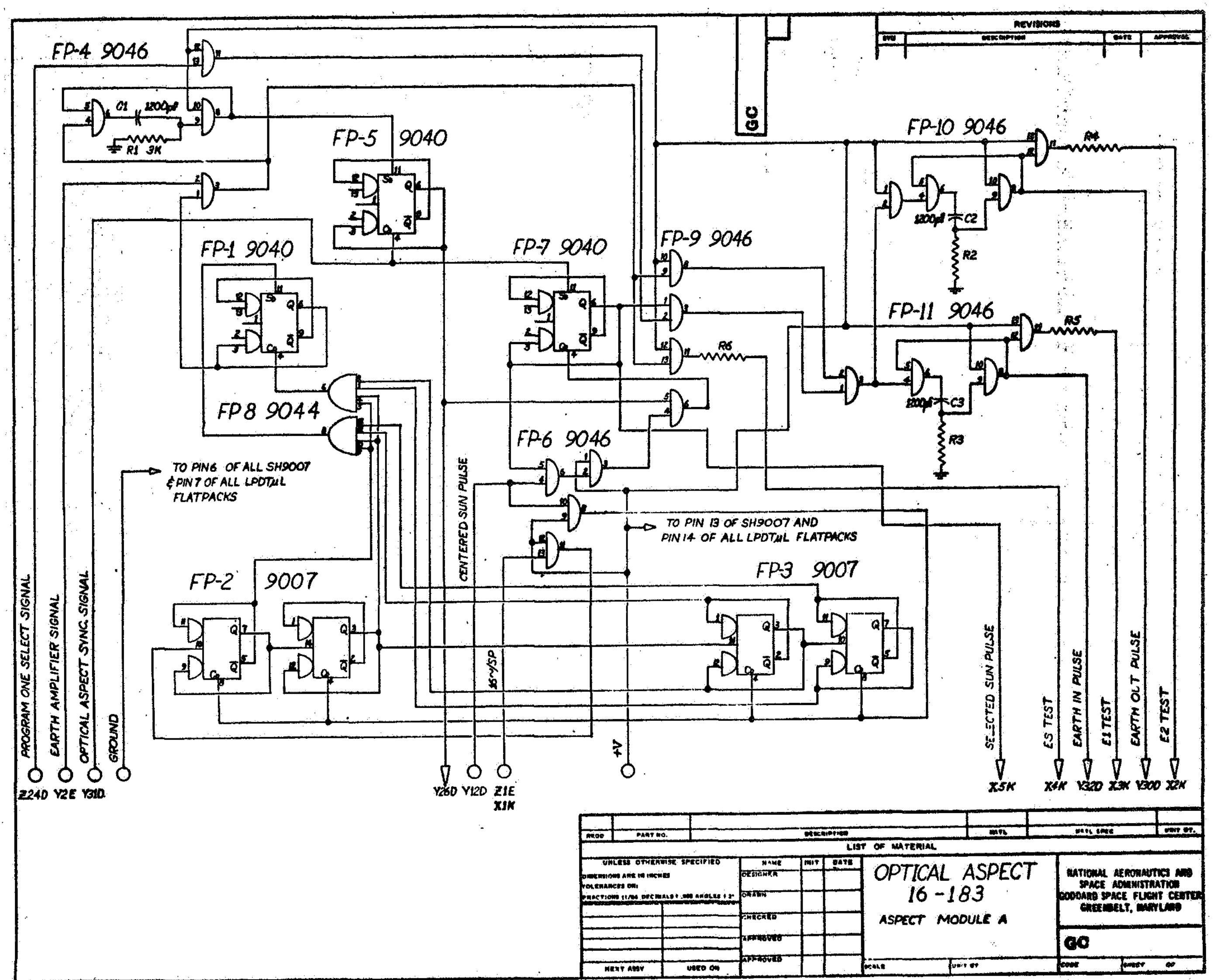

Figure 24. Optical Aspect 16-183 Aspect Module A

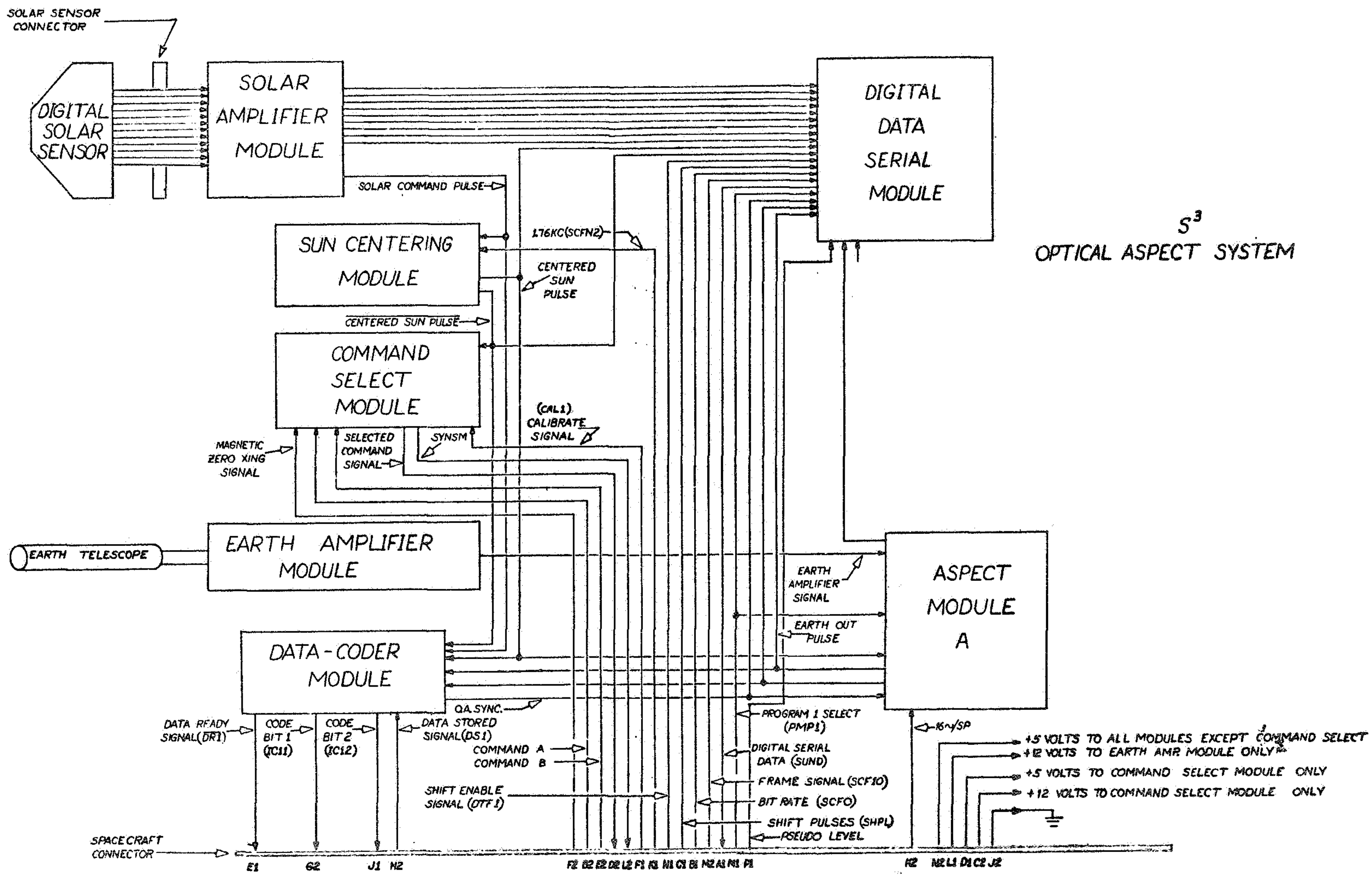

Figure 25. Optical Aspect System

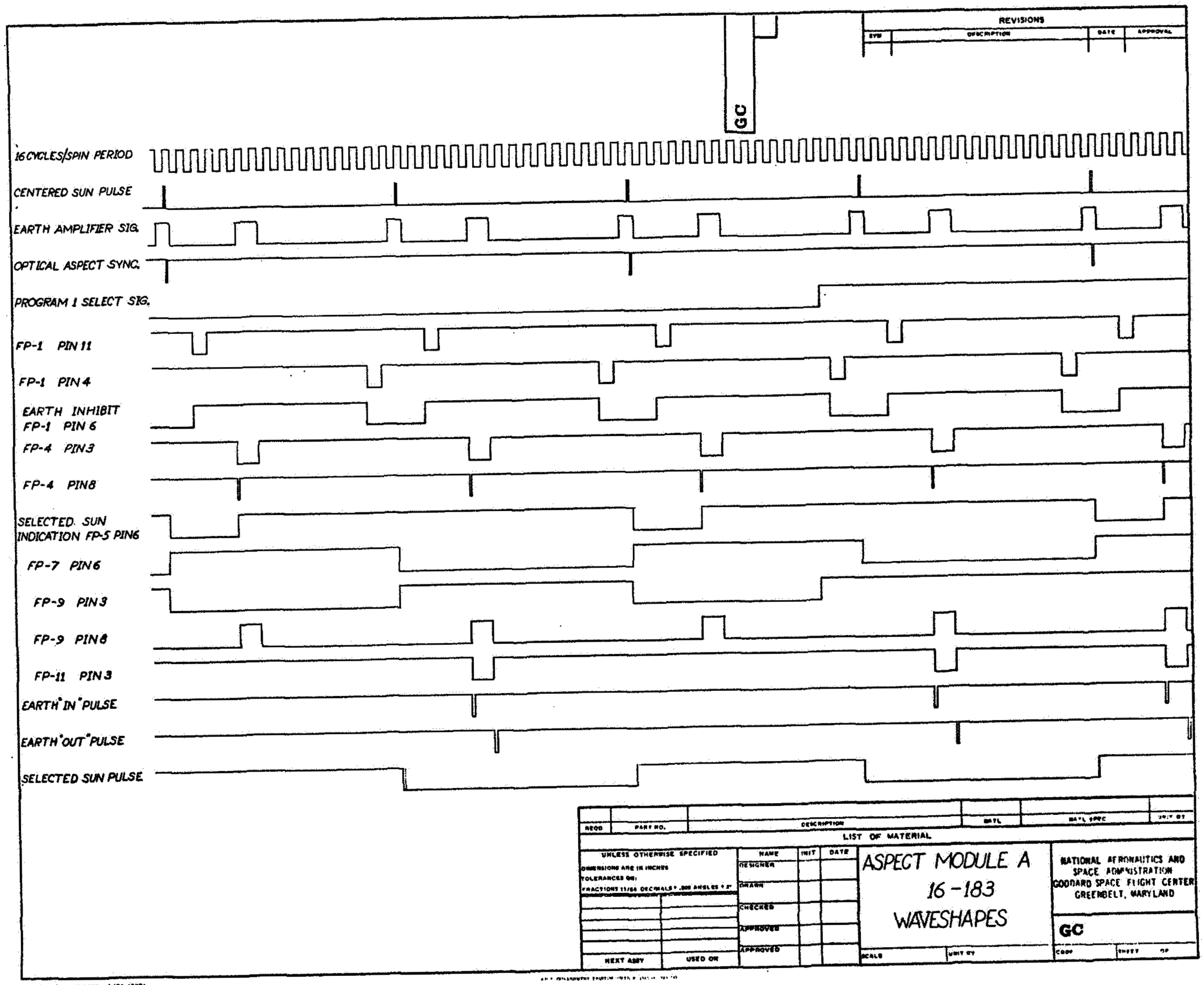

Figure 26. Aspect Module A 16-183 Waveshapes

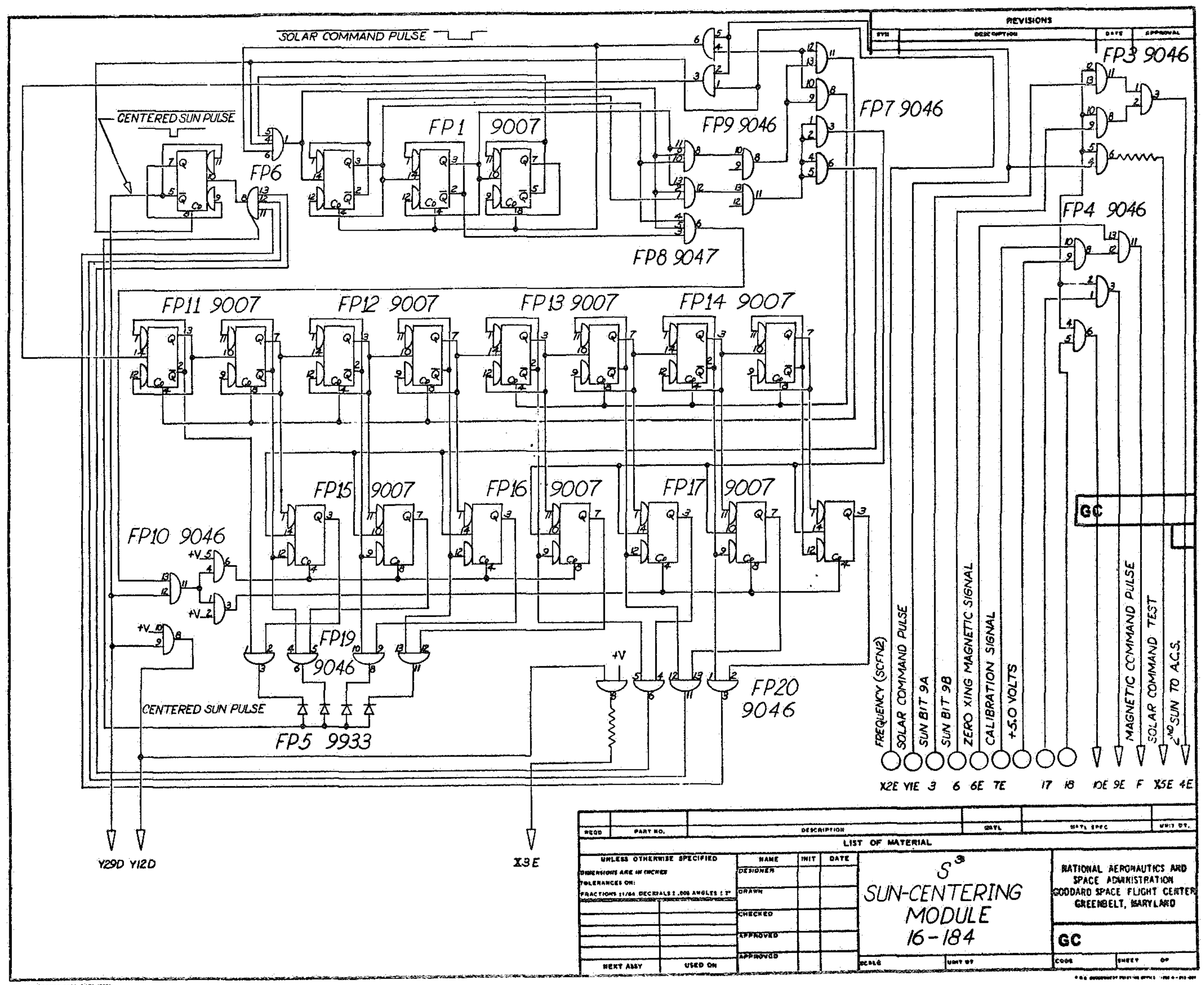

Figure 27. Sun-Centering Module 16-184